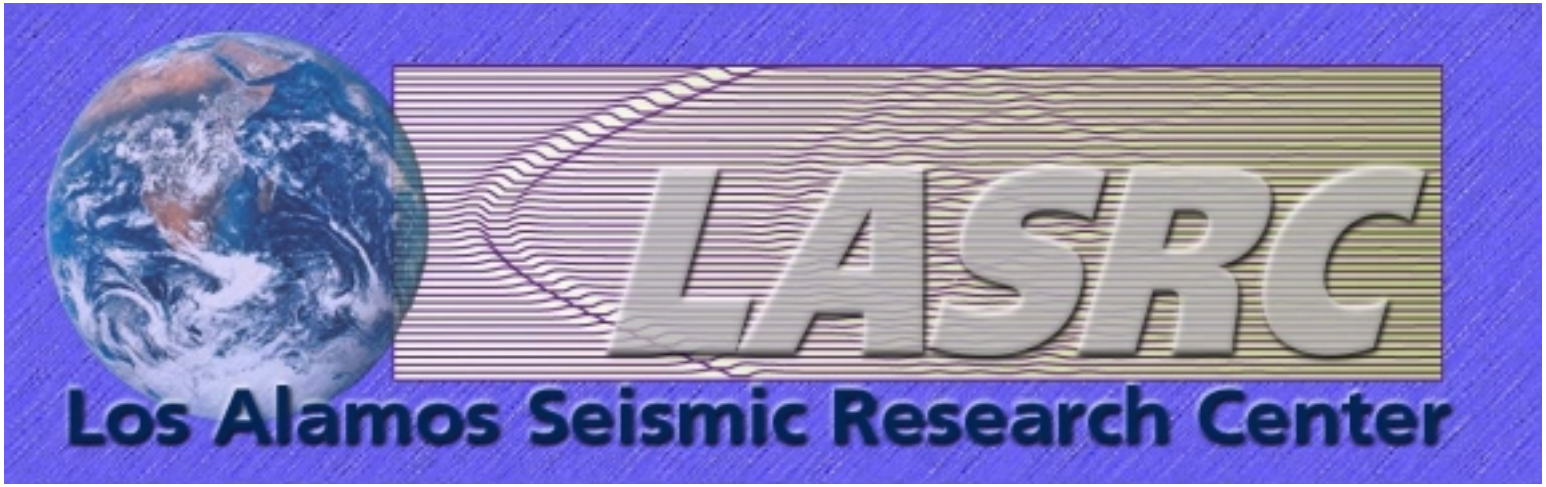
Numerical Study of Envelope Broadening in Random Media

by

Michael Fehler[1], Lian-Jie Huang[1], and Haruo Sato[2]

[1] Los Alamos Seismic Research Center, Los Alamos National Laboratory, NM 87545 USA

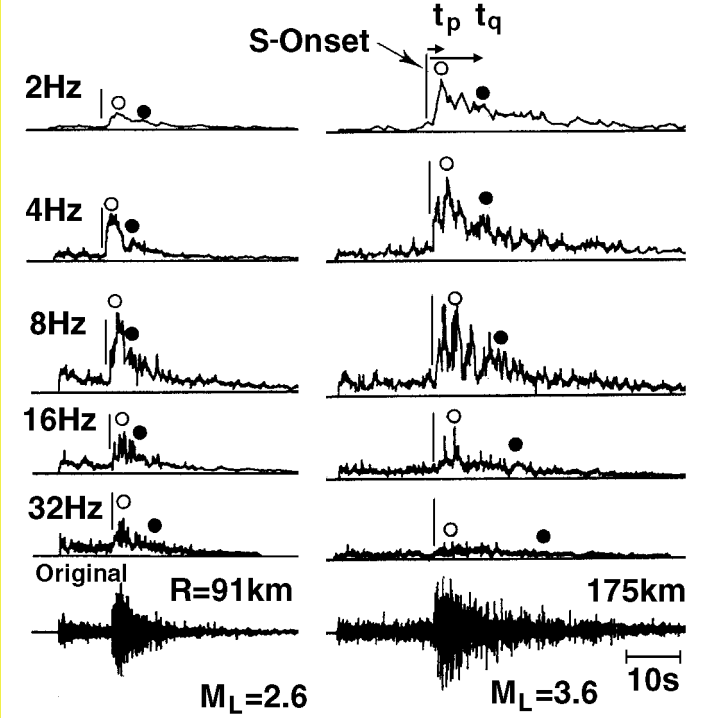
[2] Graduate School of Science, Tohoku University , Aoba-ku, Sendai-shi, 980-8578, Japan



Forward Propagation and Envelope Broadening

Regional seismograms of deep earthquakes in Japan have long coda duration due to multiple forward scattering

Lg waveforms influenced by multiple forward scattering



Seismograms from Deep Earthquakes in Japan

Los Alamos Seismic Research Center

Model for Multiple Forward Scattering

Forward scattering dominates when characteristic length of variation in medium velocity (or density) is > wavelength of waves

Can model with ***Parabolic Wave Equation*** which neglects the second derivative in the wave equation with respect to the global ray direction

Split-Step Fourier Modeling (Stoffa, et al. 1990)

Extended Local Rytov Approximation (Huang et al, 1999)

Markov Approximation

Objectives

Investigate envelope broadening using numerical (Rytov and Finite Difference) modeling

Compare results obtained using Markov Approximation with numerical results obtained using finite difference and approximate method based on Rytov Approximation.

Investigate variations in waveforms obtained from various realizations of random media

Investigate differences obtained using various modeling approaches

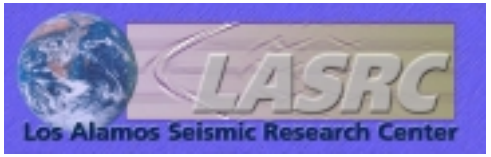
Previous Numerical Modeling of Wave Propagation in Random Media

Frankel and Clayton (1986, *JGR*)

focused on near-source region (backscattering)

Ikelle, Young and Daube (1993, *Geophysics*)

Focused on effects of anisotropy in random media characterization



Random Media Models

- Choose Model Where Forward Scattering Dominates
- Propagation to Regional Distances (>100 km)
- Write velocity as $V(\mathbf{x}) \equiv V_0 + \delta V(\mathbf{x}) = V_0(1+\xi(\mathbf{x}))$

  $\xi(\mathbf{x})$ is fractional fluctuation of wave velocity

  V_0 is chosen so that $V_0 = \langle V(\mathbf{x}) \rangle$ and $\langle \xi(\mathbf{x}) \rangle = 0$

Random Media Models

Autocorrelation function (ACF) of the medium

$$R(\mathbf{x}) \equiv \langle \xi(\mathbf{y}) \xi(\mathbf{y}+\mathbf{x}) \rangle$$

Magnitude of fractional fluctuation is mean square (MS) fractional fluctuation:

$$\varepsilon^2 \equiv R(0) = \left\langle \xi(\mathbf{x})^2 \right\rangle$$

Choose Gaussian ACF

$$R(\mathbf{x}) = R(r) = \varepsilon^2 \, e^{-\frac{x^2}{a_x{}^2}} \, e^{-\frac{z^2}{a_z{}^2}}$$

where a_x and a_z are correlation distances.

Medium characterized by

RMS fractional fluctuation ε

Correlation distance a

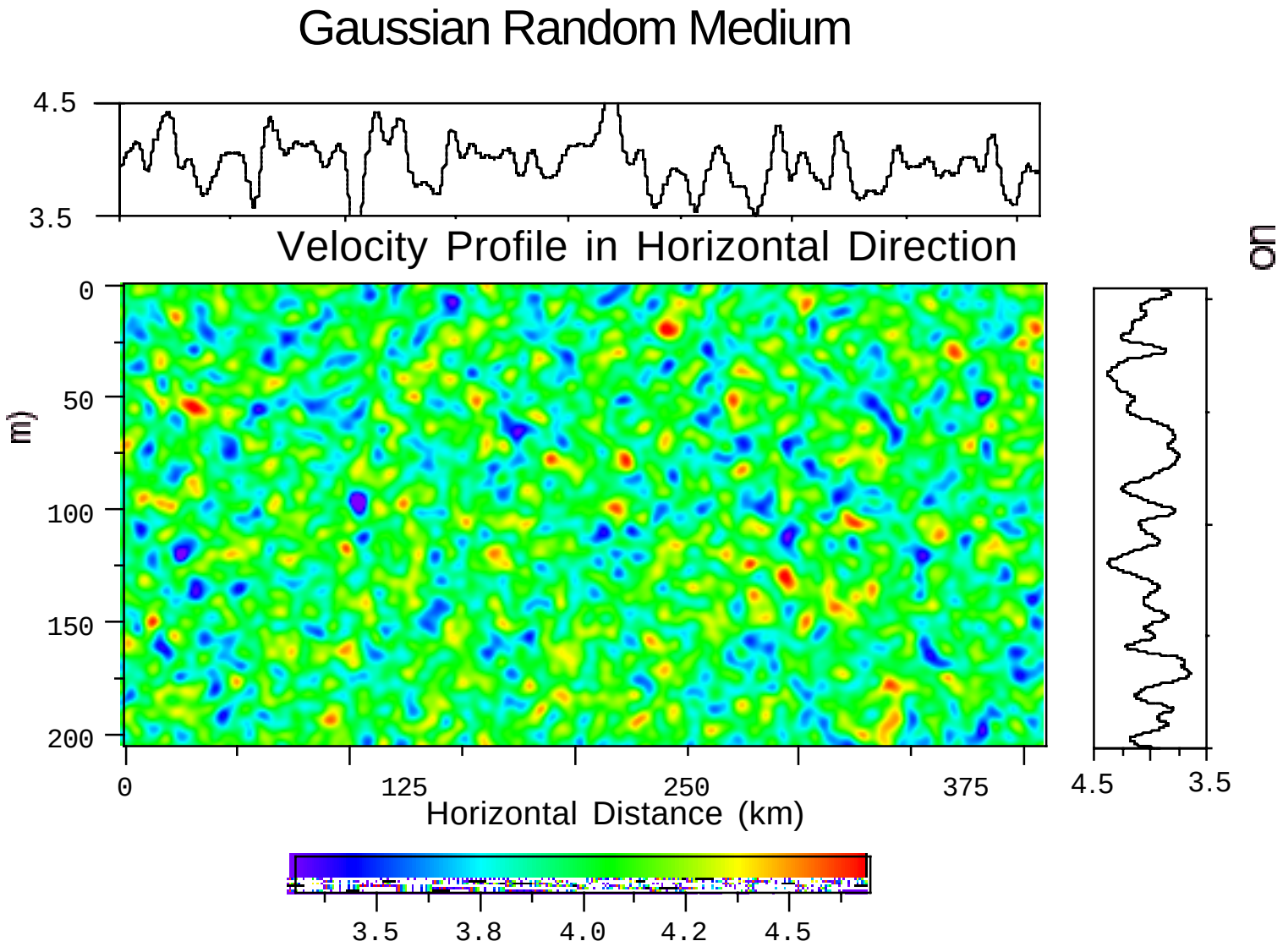
We choose $\varepsilon = .05$

$a = 5$ km

$V_0 = 4.$ km/s

Source: Ricker time history with 2Hz dominant frequency

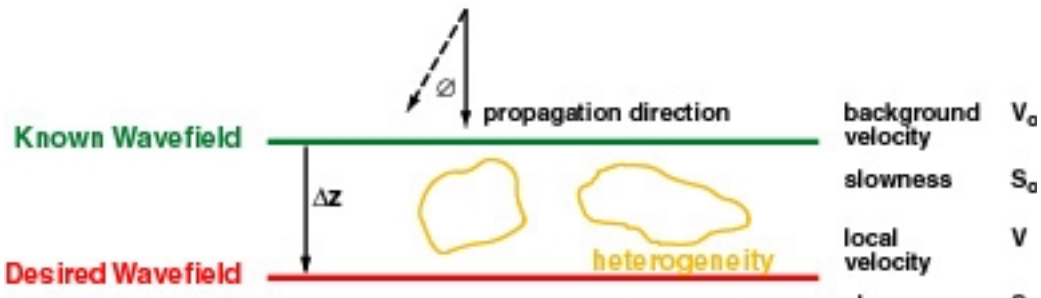
Wavelength in background medium is 2 km


Gaussian Random Medium
4.5
3.5
Velocity Profile in Horizontal Direction
0
50
100
150
200
m)
0
125
250
375
Horizontal Distance (km)
4.5
3.5
3.5
3.8
4.0
4.2
4.5

Markov Approximation

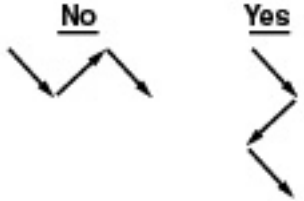
- Method to calculate mean wavefield for propagation in random media when forward scattering dominates
- Lee and Jakopi (1975) Sreenivasiah et al. (1976) used to calculate envelope for plane waves incident on 3D random media
- Sato (*JGR*,1989) used to study envelope broadening
- Here, calculated for point source in 2D random media

One-Way Wave Propagation



Reverberations not modeled

Modeling is fast and local (less memory)



Our methods use

Local Born $P = P_0 + P_S$

Local Rytov $P = P_0 e^{\phi_S}$ ϕ_S complex

Approximation

⇒ Background Velocity V_0 can be unique for each layer

Local Rytov Fourier Method

(Huang et al. ,*Geophysics* **64**, 1535-1545, 1999)

Wave Equation

$$\left(\Delta - \frac{1}{V(\mathbf{x})^2}\partial_t^{\,2}\right)u(\mathbf{x},t) = 0$$

$$\Delta = \partial^2/\partial x^2 + \partial^2/\partial z^2$$

Using

$$V(\mathbf{x}) \equiv V_0 + \delta V(\mathbf{x}) = V_0\left(1+\xi(\mathbf{x})\right)$$

$$|\xi| << 1$$

We get

$$\left(\Delta - \frac{1}{V_0^{\,2}}\partial_t^{\,2}\right)u(\mathbf{x},t) + \frac{2}{V_0^{\,2}}\xi(\mathbf{x})\partial_t^{\,2}u(\mathbf{x},t) = 0$$

Frequency Domain

$$\left(\Delta + \frac{\omega^2}{V_0^{\,2}}\right)u(\mathbf{x},\omega) = 2\frac{\omega^2}{V_0^{\,2}}\xi(\mathbf{x})u(\mathbf{x},\omega)$$

Local Rytov Fourier Method (cont.)

Solution of form $u(\mathbf{x},\omega) = e^{\phi_0(\mathbf{x},\omega)+\phi_s(\mathbf{x},\omega)}$

Homogeneous propagation $u_0(\mathbf{x},\omega) = e^{\phi_0(\mathbf{x},\omega)}$

Extrapolate wavefield in primary propagation direction

$$u(x,z_{i+1};\omega) = u_0(x,z_{i+1};\omega)e^{\phi_S(x,z_{i+1;};\omega)}$$

$$u_0(x,z_{i+1};\omega) = F_{k_x}^{-1}\left\{e^{ik_z\Delta z}F_x\ \{u(x,z_i,\omega\}\right\}$$

Local Rytov Fourier Method (cont.)

Scattering term: $\phi_S(x, z_{i+1}; \omega) \equiv \psi(x, z_{i+1}; \omega) / u_0(\mathbf{x}_T, z_{i+1}; \omega)$

$$\psi(x, z_{i+1}; \omega) = F_{k_x}^{-1}\left\{\frac{k_0}{k_z} e^{ik_z \Delta z} F_x\left\{i\omega \Delta z \Delta s(x, z_i) u(x, z_i; \omega)\right\}\right\}$$

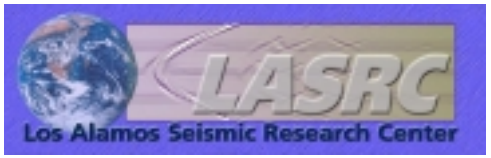
Slowness perturbation: $\Delta s(x, z_i) = -\xi(x, z_i) / V(x, z_i)$

Wavenumbers: $k_z = \sqrt{k_0^2 - k_x^2}\ , \ \ k_0 = \omega / V_0$

Fourier Transforms: $F_x,\ F_{k_x}^{-1}$

Finite Difference Modeling

- 2D Finite Difference Code
- 4th order in Space
- 2th Order in Time
- Holberg Coefficients to Minimize Dispersion
- Absorbing Boundaries

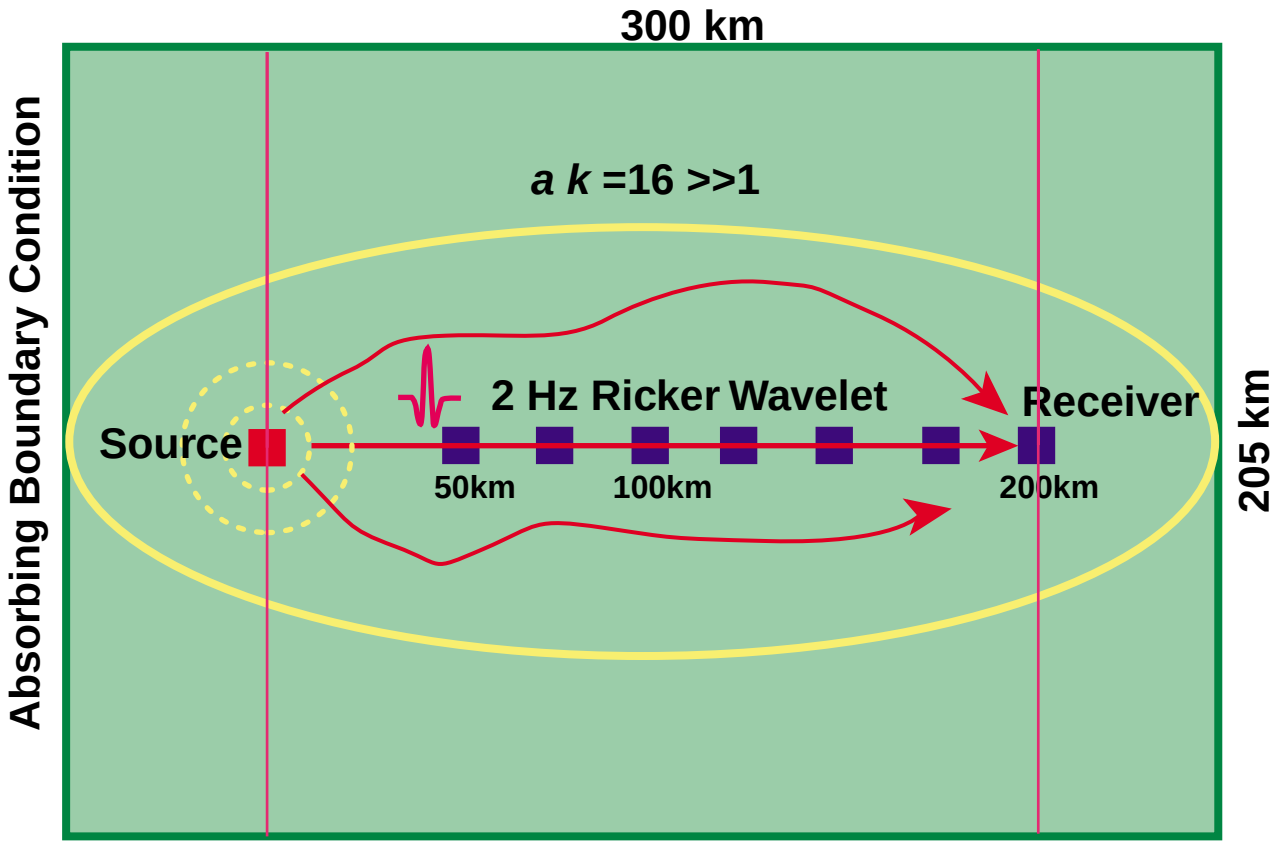


Numerical Experiment

Finite Difference: Grid Size 50m, Time Step 4 ms

Random Media of Gaussian ACF

ε= 0.05, a = 5 km, V_0 = 4 km/s



4,096 x 6,000 grid points in region of validity

Finite Difference Grid: 4,146 x 6,052

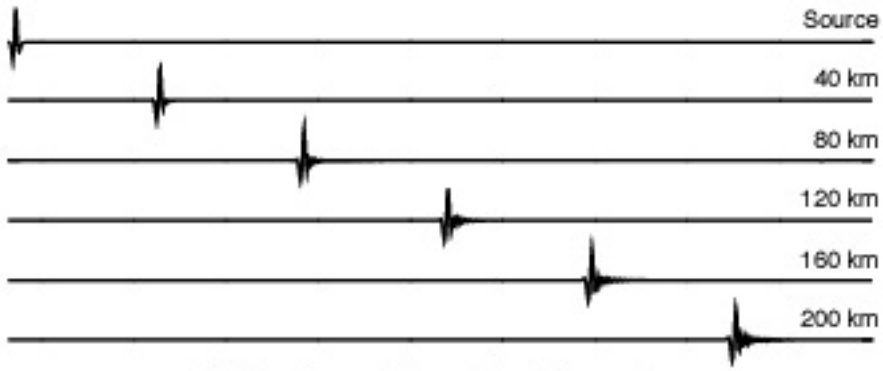
Extended Local Rytov Fourier Grid: 4,096 X 4,096

Valid up to 25s after the onset at 200 km distance

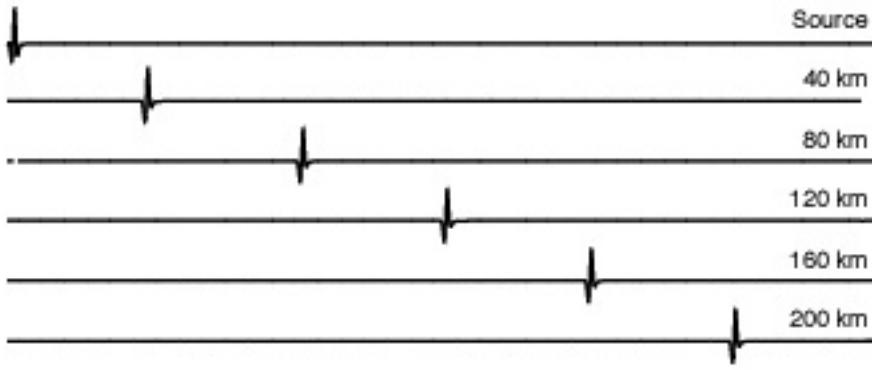
Finite Difference Seismograms in Homogeneous Media

Velocity = 4 km/s; Source: 2 Hz Ricker

Trace Length = 60 s; Trace Amplitudes Normalized

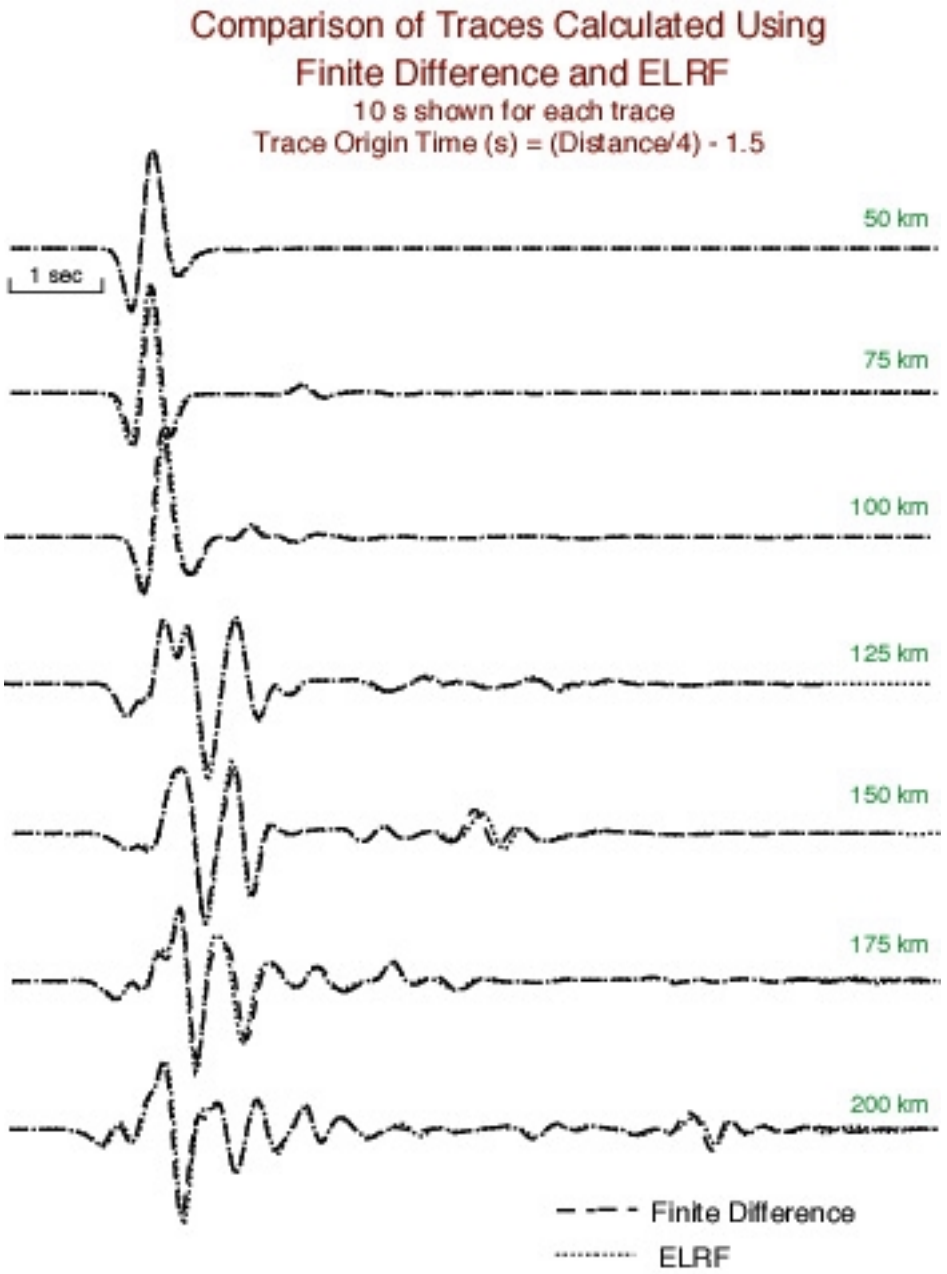


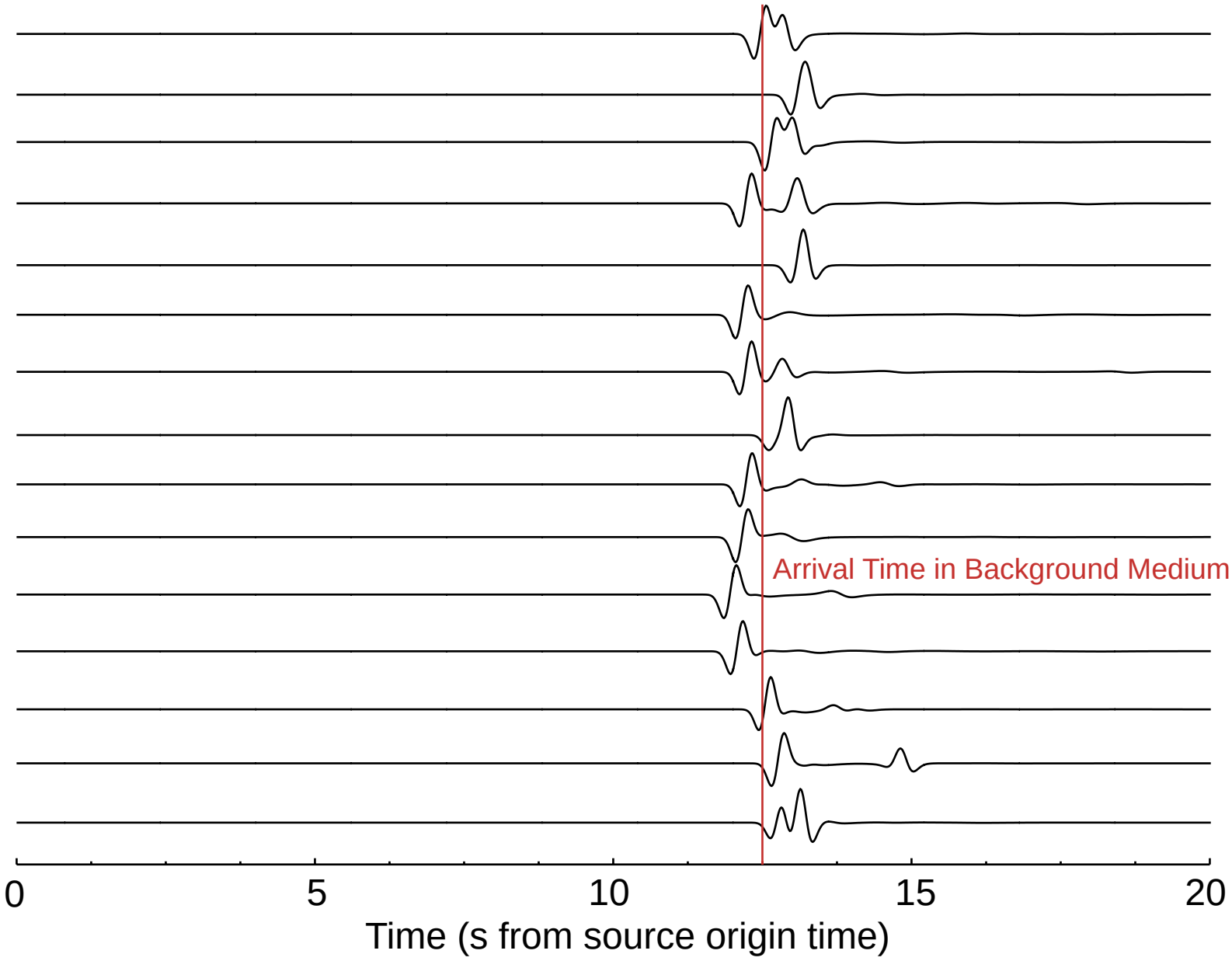
Grid Spacing = 200 m; Time Step = 16 ms

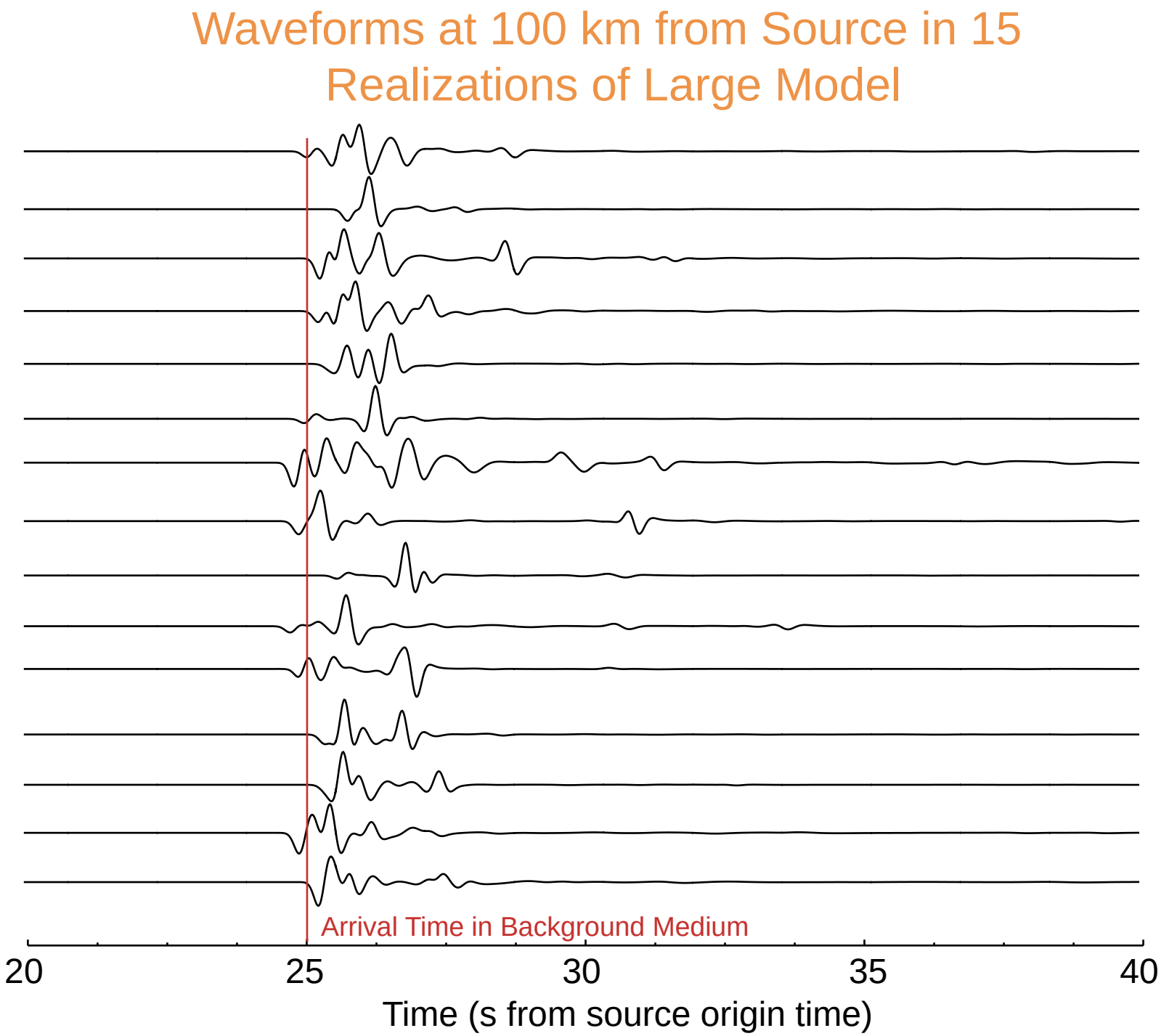


Grid Spacing = 50 m; Time Step = 4 ms

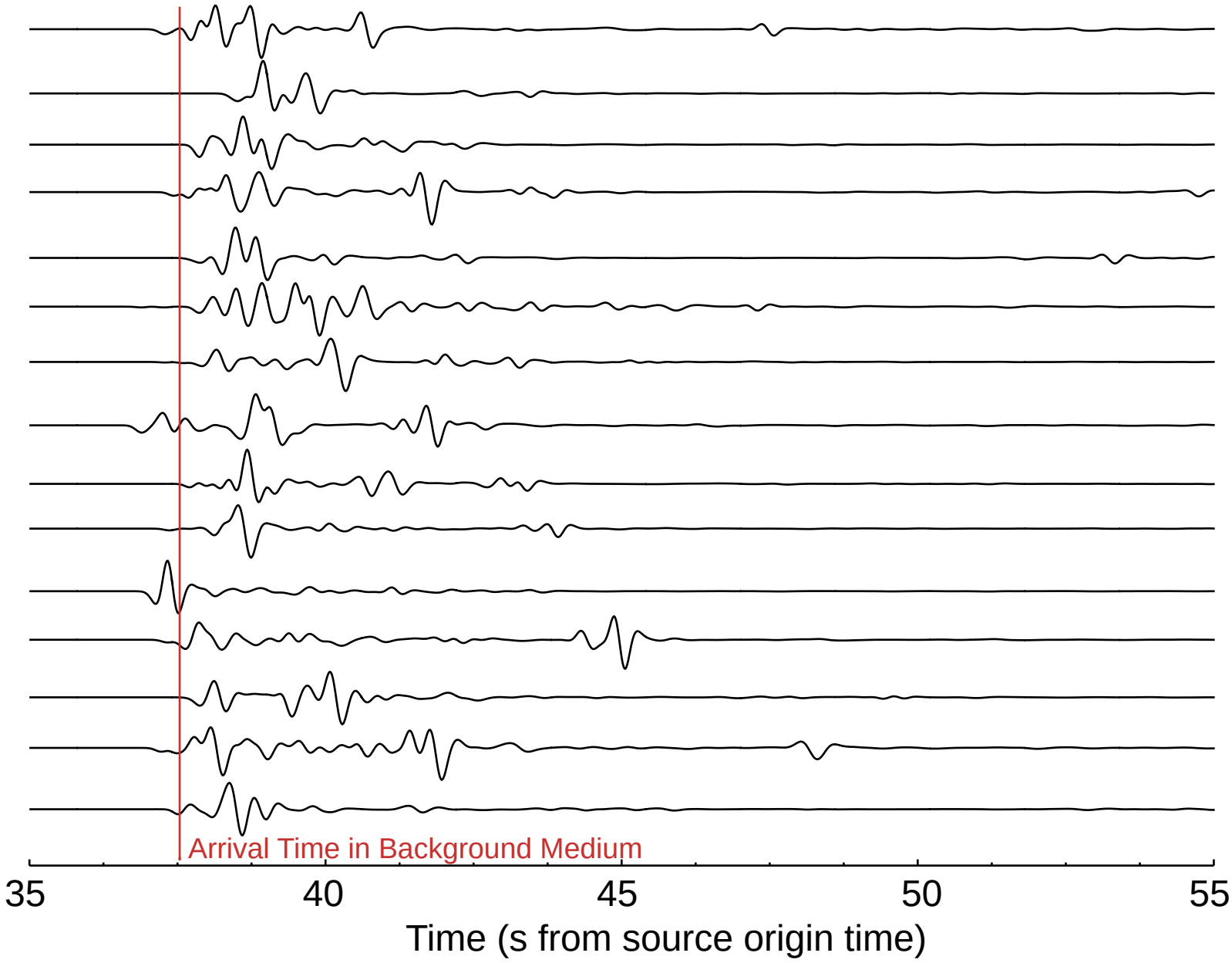
Figure 2

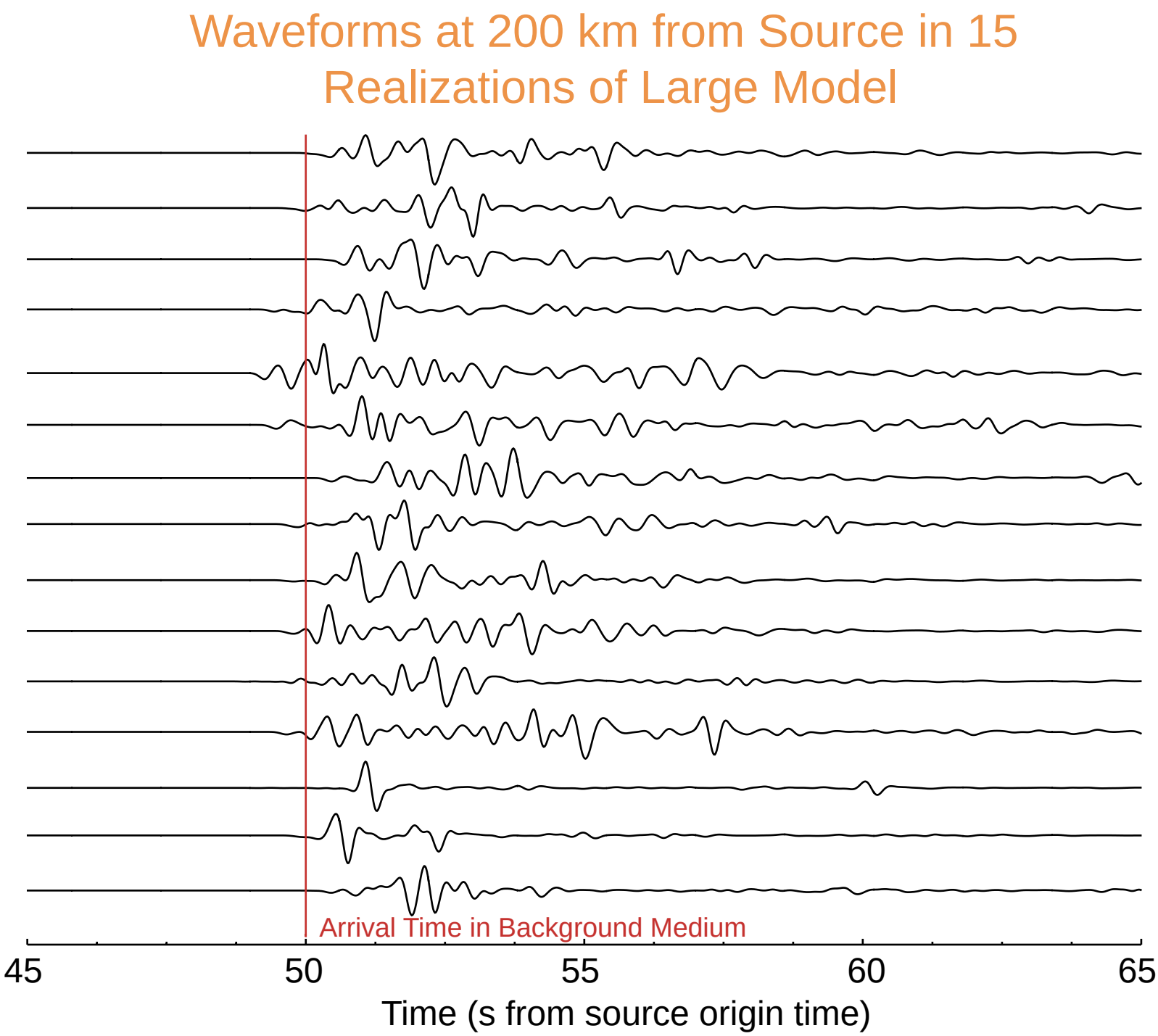

Comparison of Traces Calculated Using
Finite Difference and ELRF
10 s shown for each trace
Trace Origin Time (s) = (Distance/4) - 1.5
50 km
1 sec
75 km
100 km
125 km
150 km
175 km
200 km
Finite Difference
ELRF

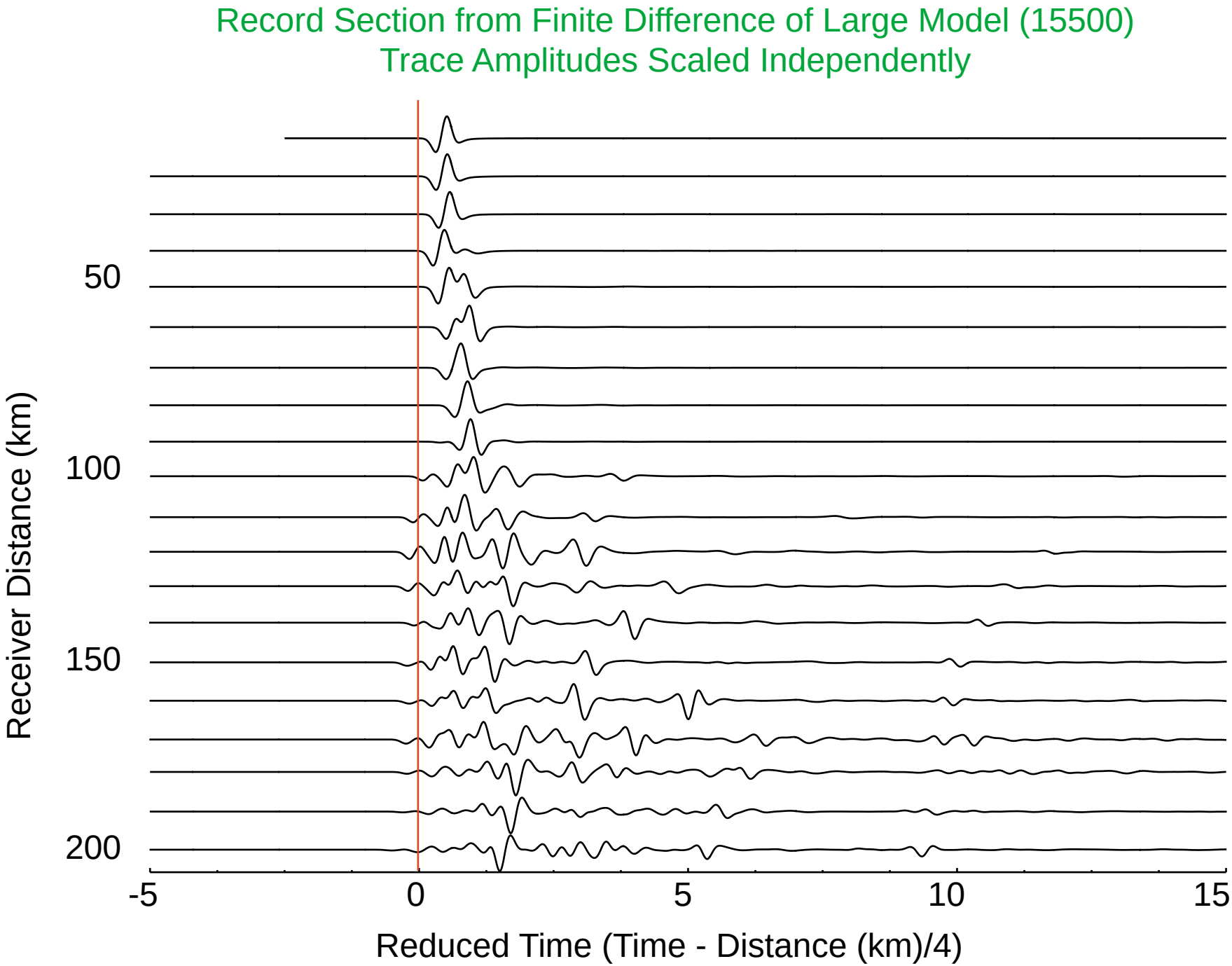

Waveforms at 50 km from Source in 15 Realizations of Large Model
Arrival Time in Background Medium
0
5
10
15
20
Time (s from source origin time)

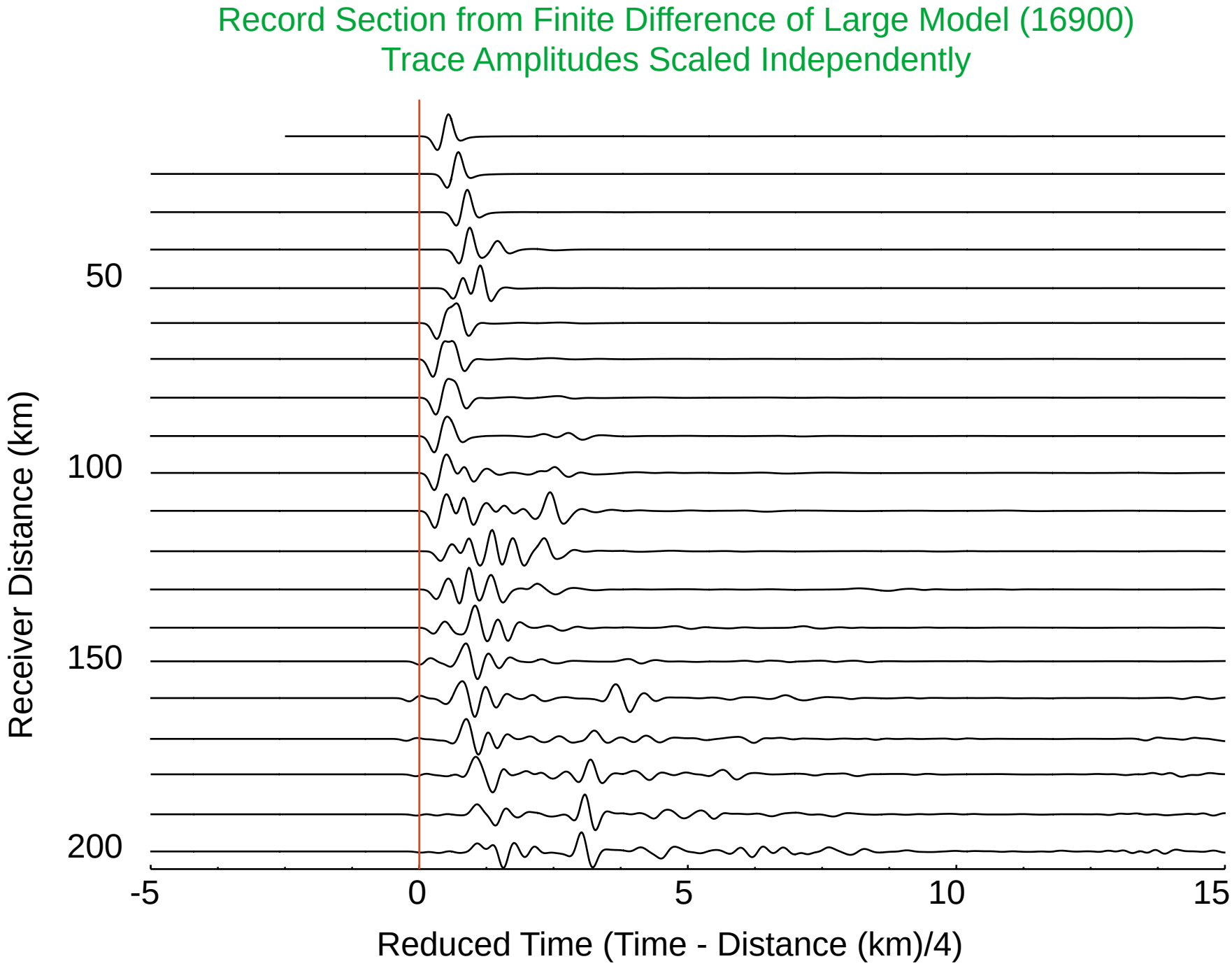

Waveforms at 100 km from Source in 15 Realizations of Large Model
Arrival Time in Background Medium
20
25
30
35
40
Time (s from source origin time)

Waveforms at 150 km from Source in 15 Realizations of Large Model



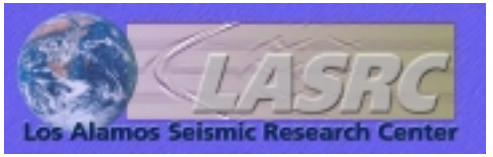

Waveforms at 200 km from Source in 15 Realizations of Large Model
Arrival Time in Background Medium
45
50
55
60
65
Time (s from source origin time)


Record Section from Finite Difference of Large Model (15500)
Trace Amplitudes Scaled Independently
Receiver Distance (km)
50
100
150
200
-5
0
5
10
15
Reduced Time (Time - Distance (km)/4)


Record Section from Finite Difference of Large Model (16900)
Trace Amplitudes Scaled Independently
Receiver Distance (km)
50
100
150
200
-5
0
5
10
15
Reduced Time (Time - Distance (km)/4)

Ensemble Average

- Calculate Waveforms for Propagation in many realizations of media
- Find mean envelope Shape
  - Sum Square Traces from each realization
  - Smooth over .32s
  - Plot square root of result



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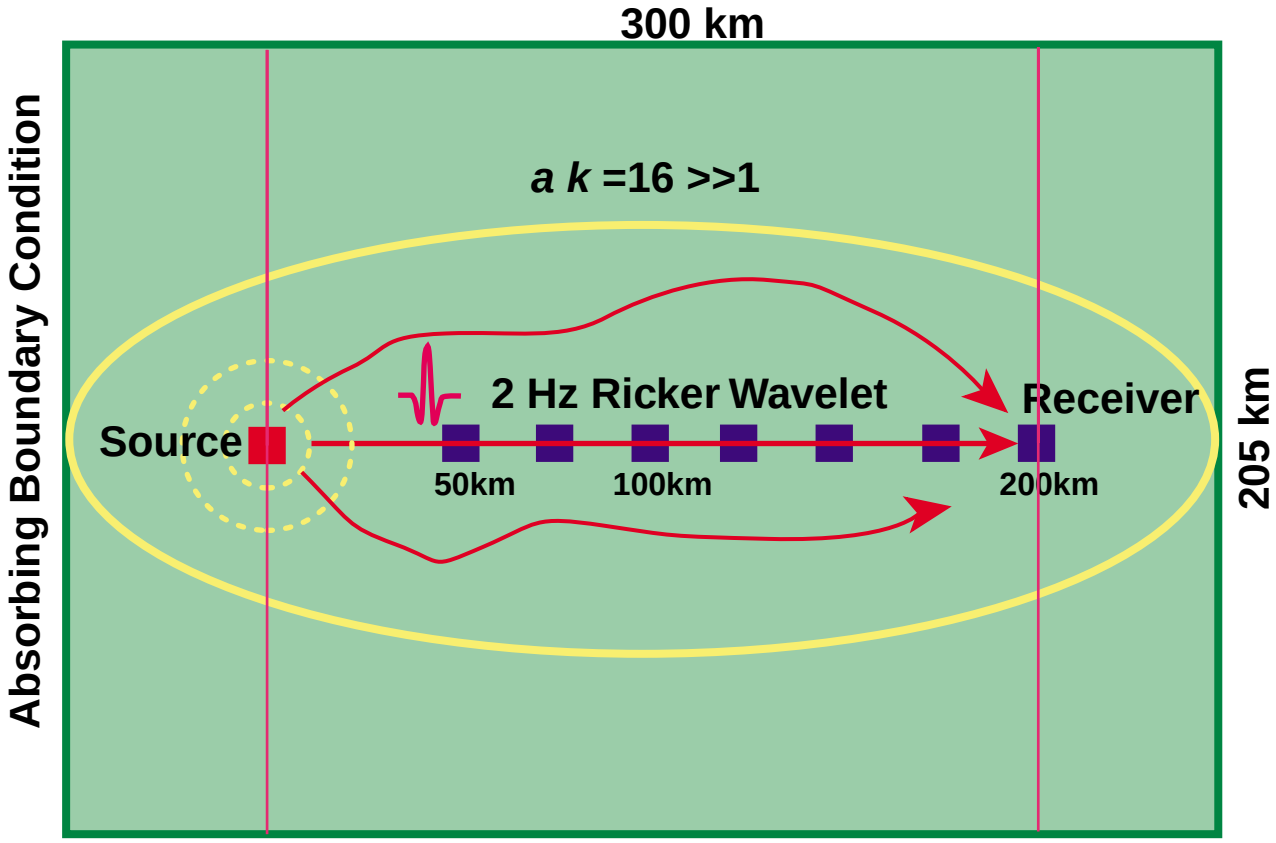

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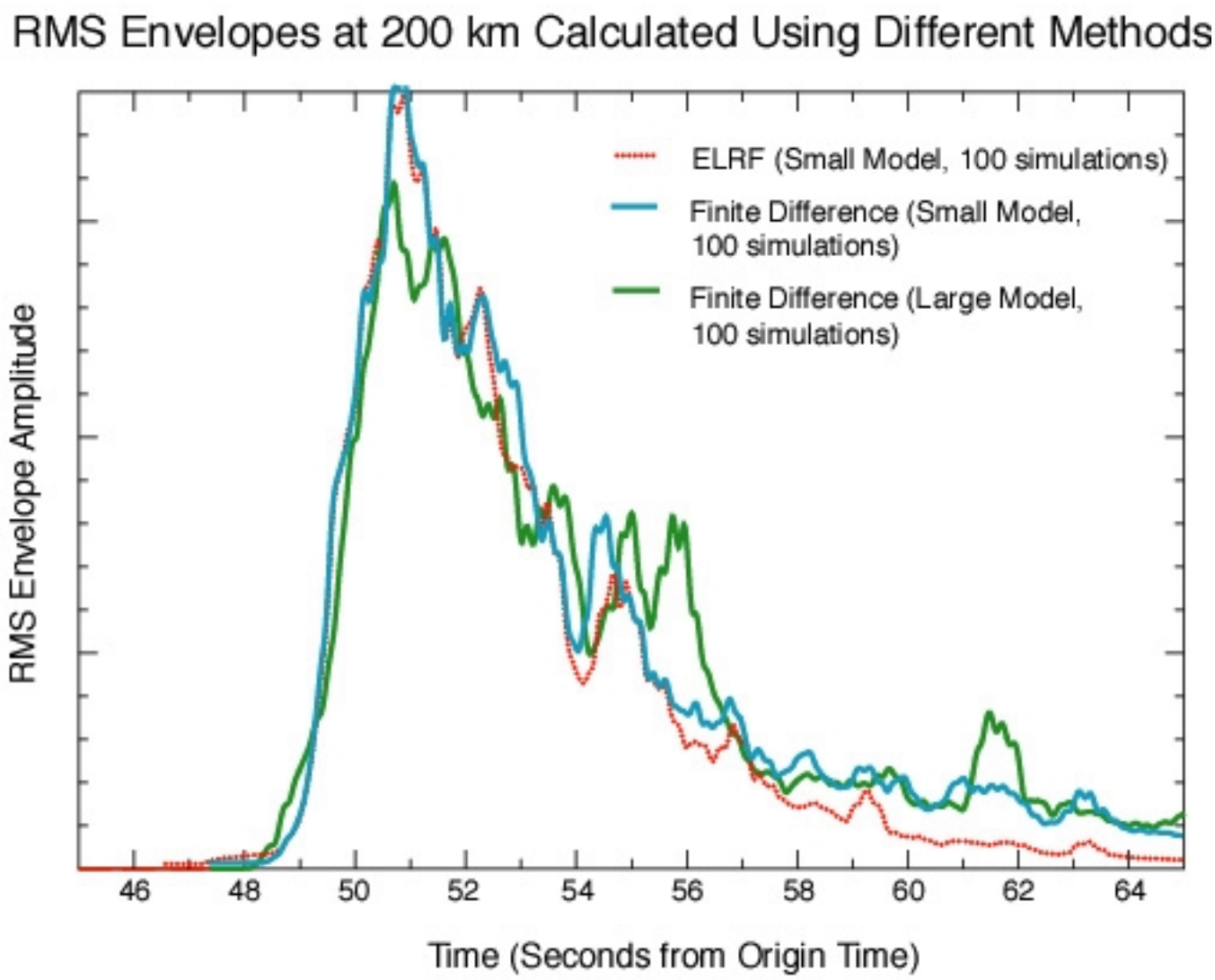
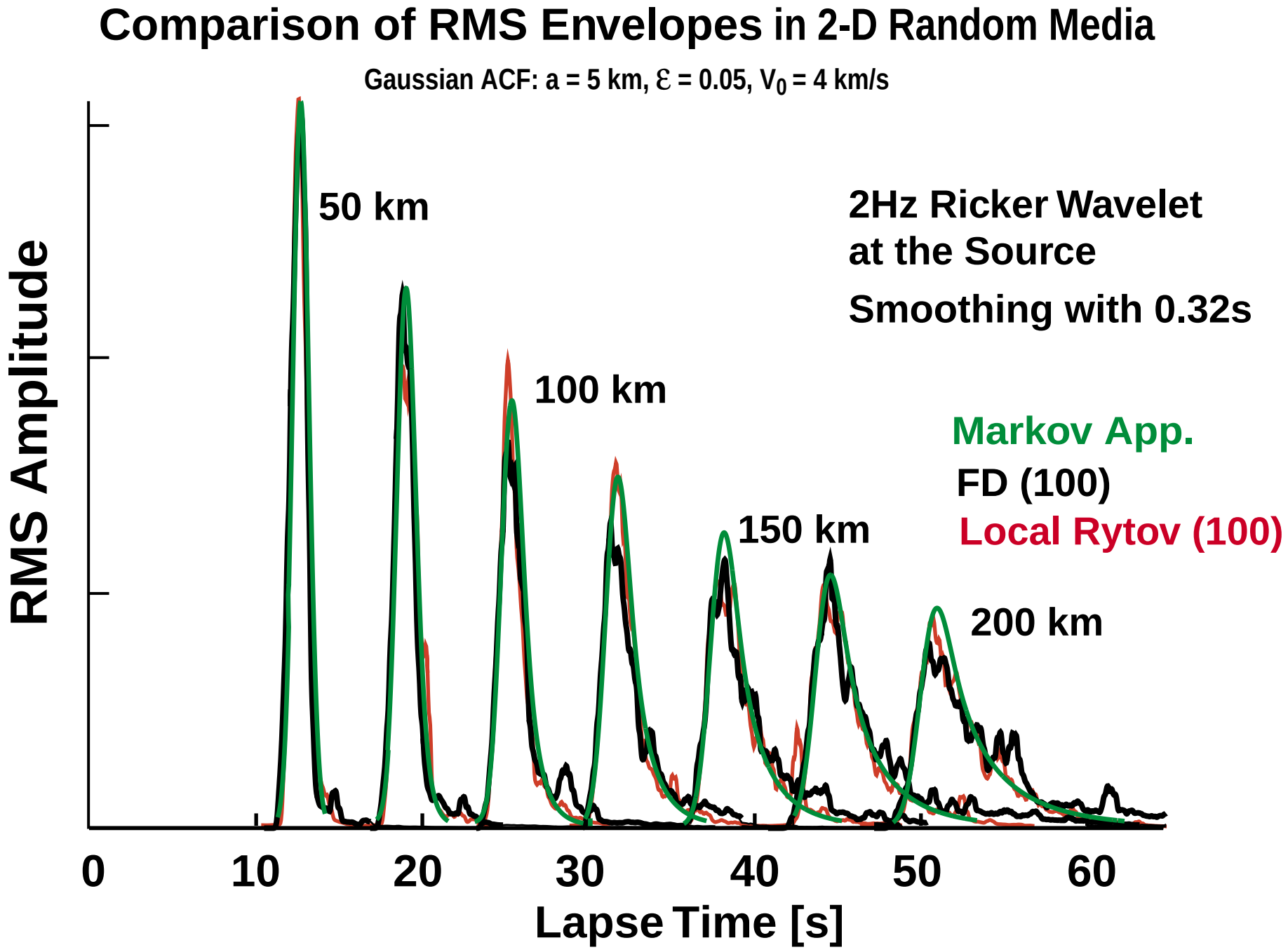
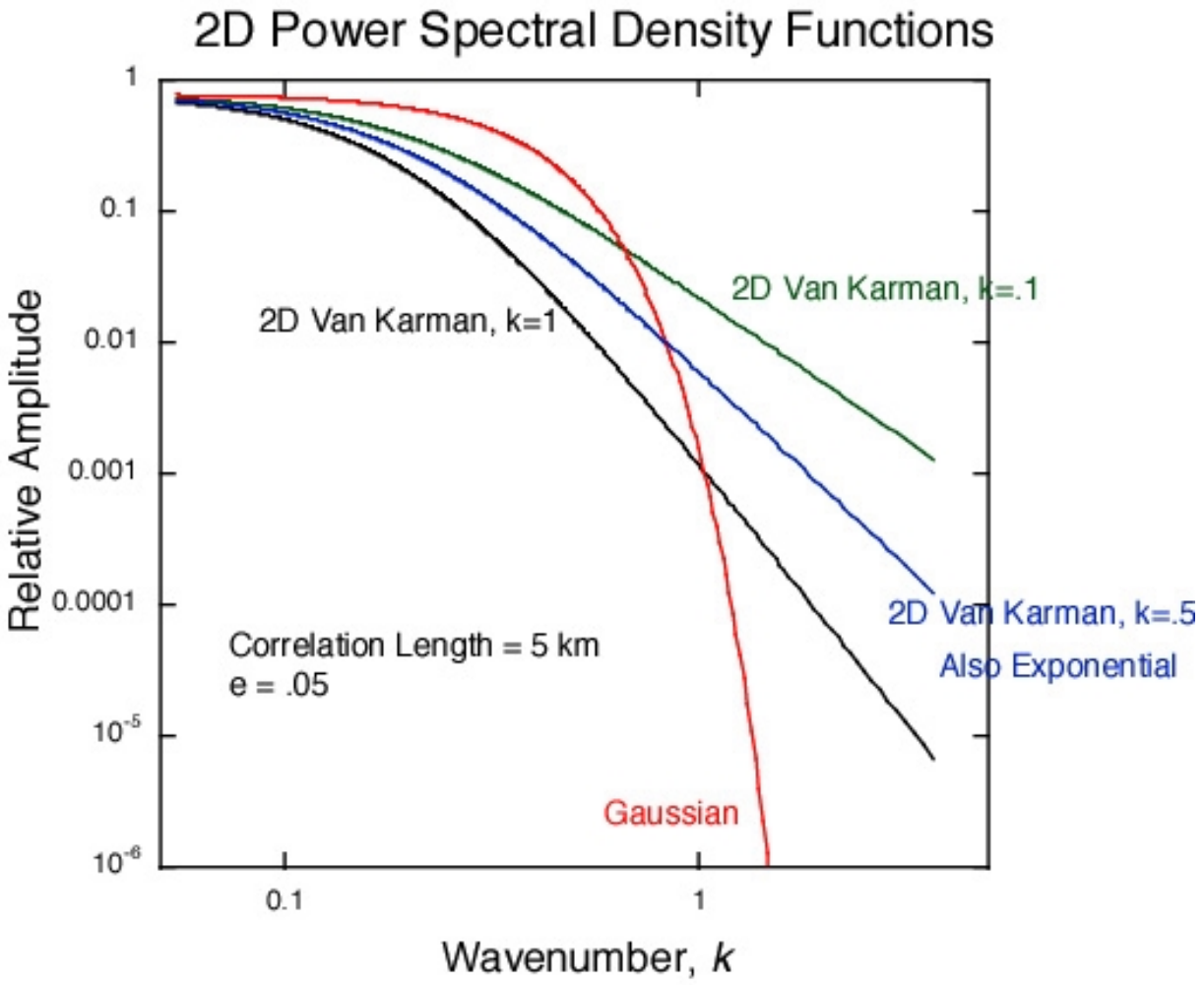
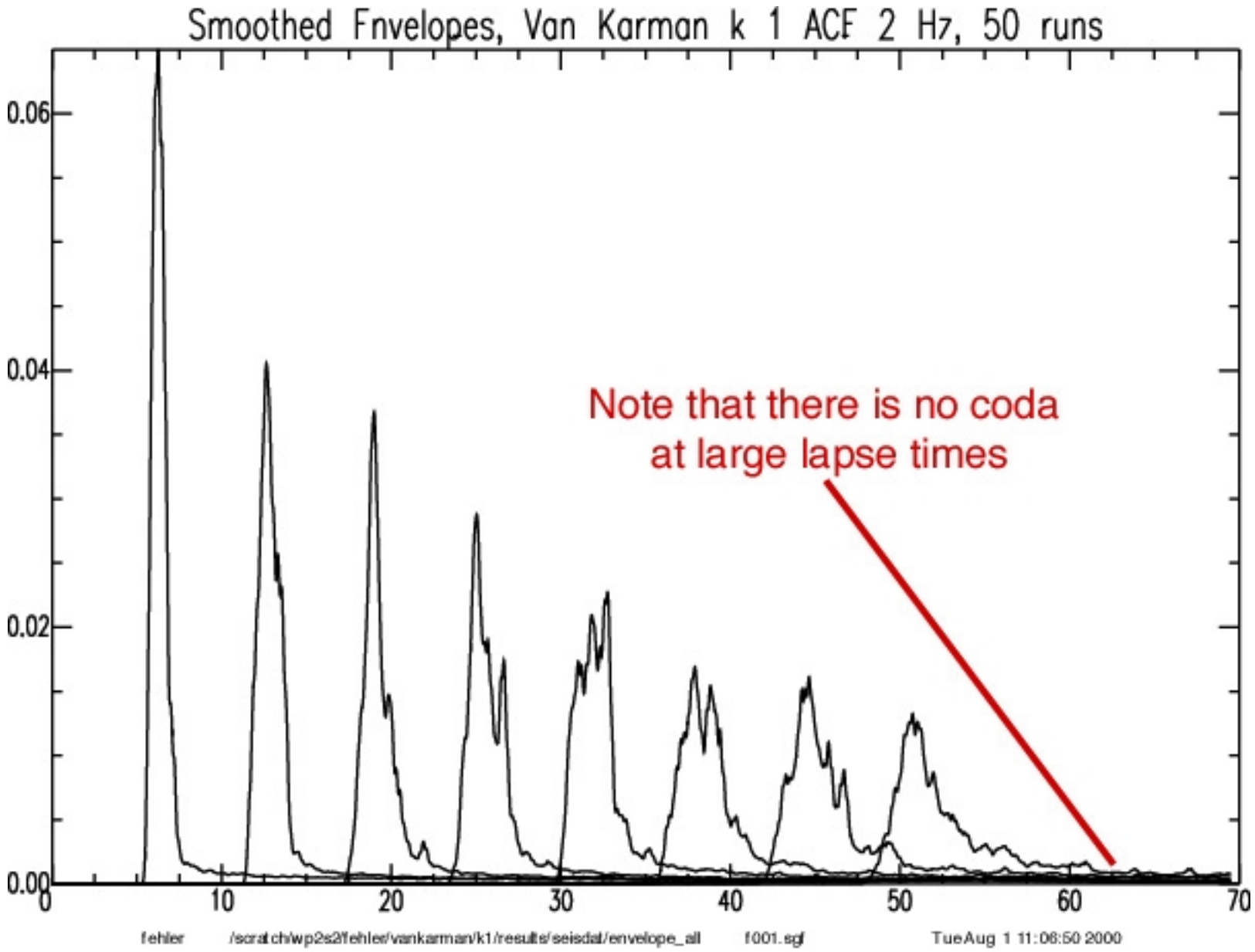
Valid up to 25s after the onset at 200 km distance

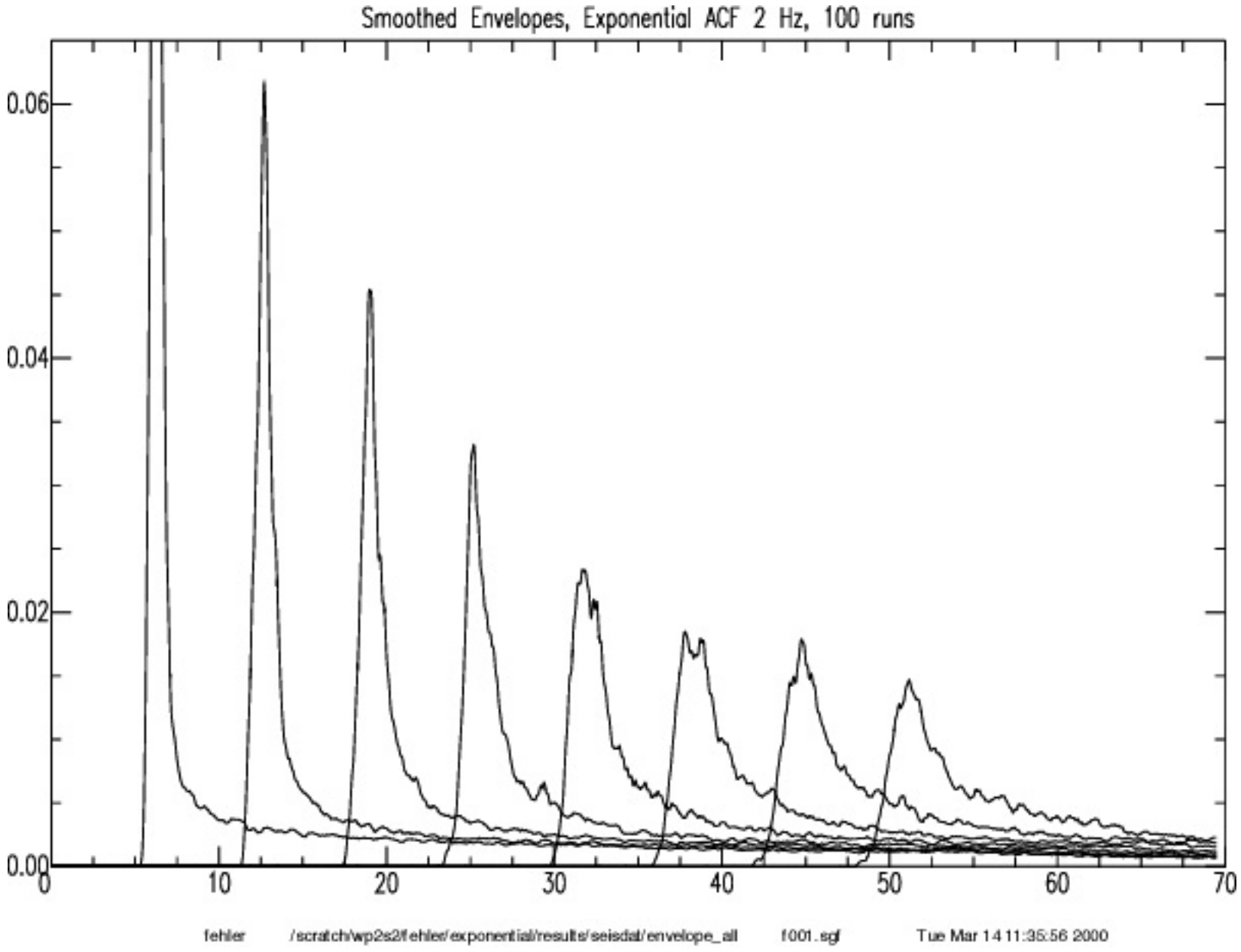

RMS Envelopes at 200 km Calculated Using Different Methods
ELRF (Small Model, 100 simulations)
Finite Difference (Small Model, 100 simulations)
Finite Difference (Large Model, 100 simulations)
RMS Envelope Amplitude
46
48
50
52
54
56
58
60
62
64
Time (Seconds from Origin Time)

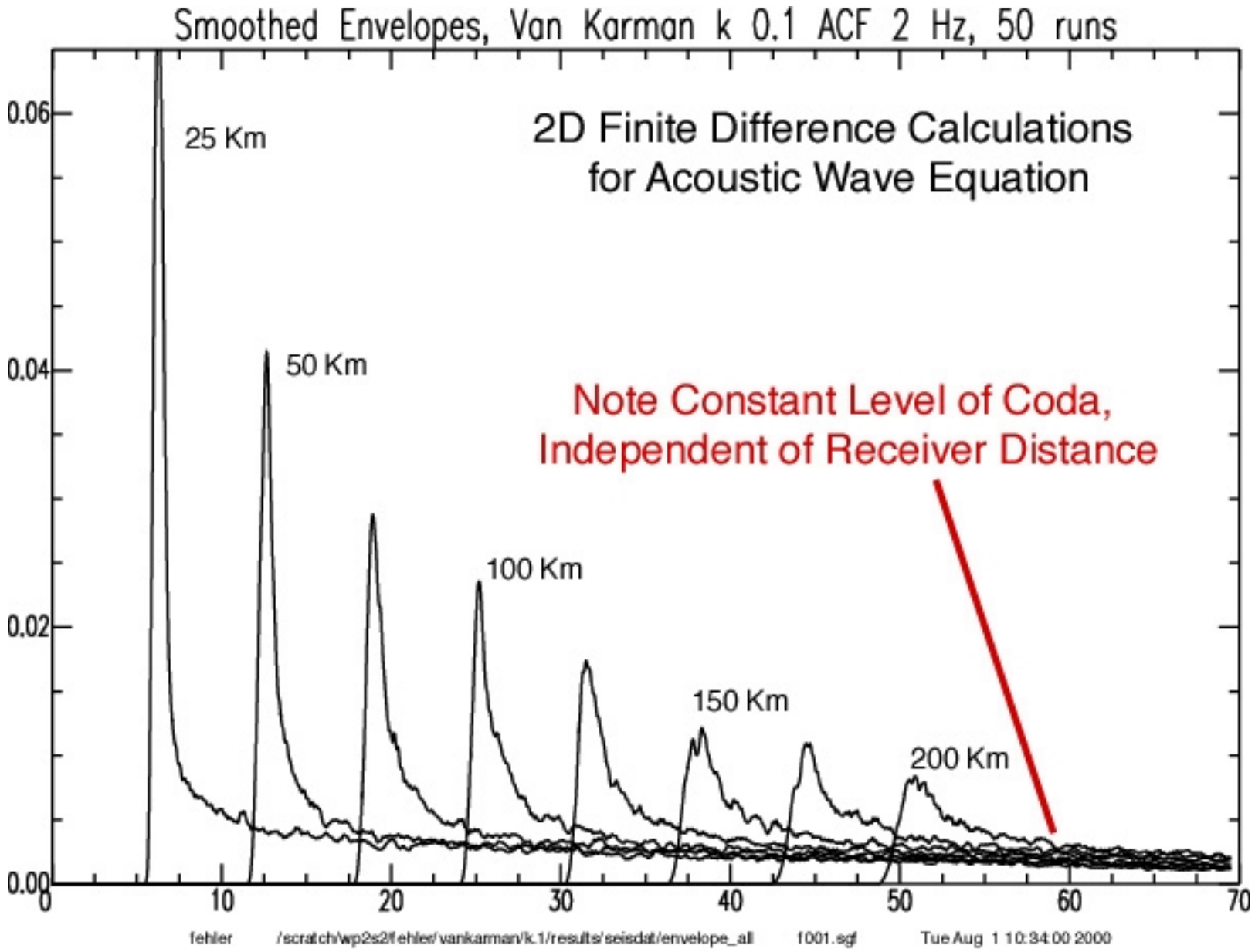

Comparison of RMS Envelopes in 2-D Random Media
Gaussian ACF: a = 5 km, ε = 0.05, V0 = 4 km/s
RMS Amplitude
50 km
100 km
150 km
200 km
2Hz Ricker Wavelet
at the Source
Smoothing with 0.32s
Markov App.
FD (100)
Local Rytov (100)
0
10
20
30
40
50
60
Lapse Time [s]

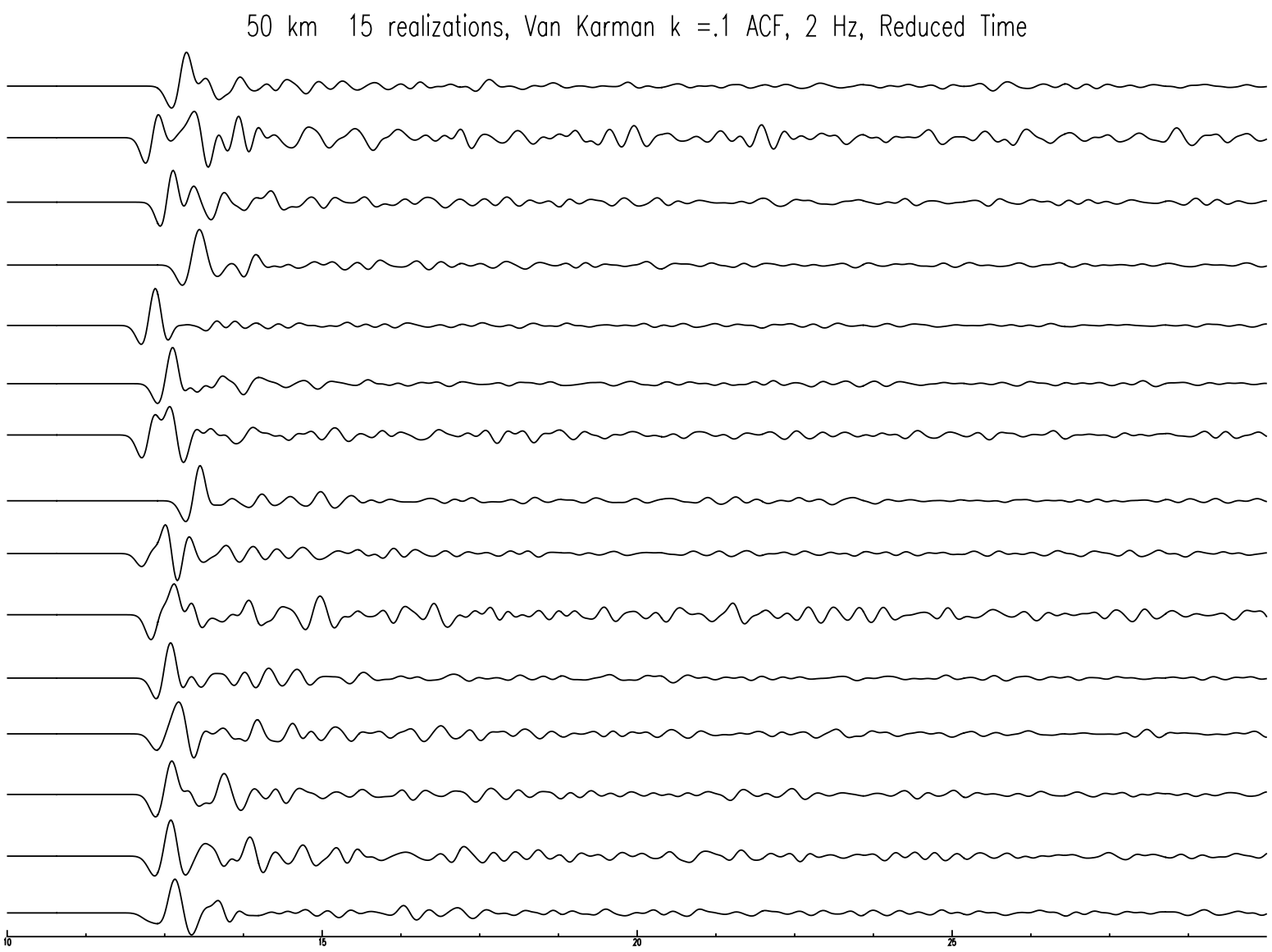
Comparisons with Different Correlation Functions

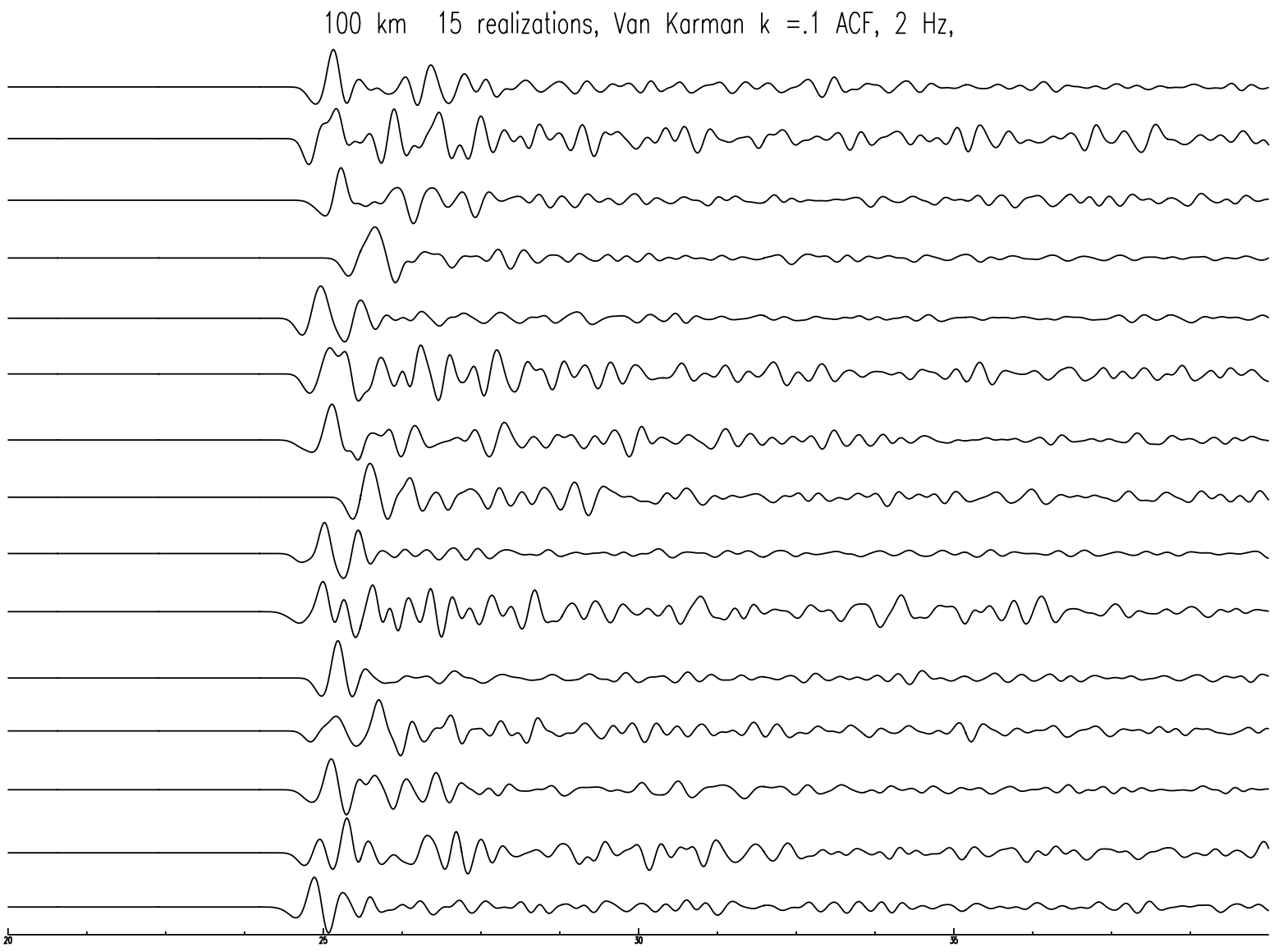


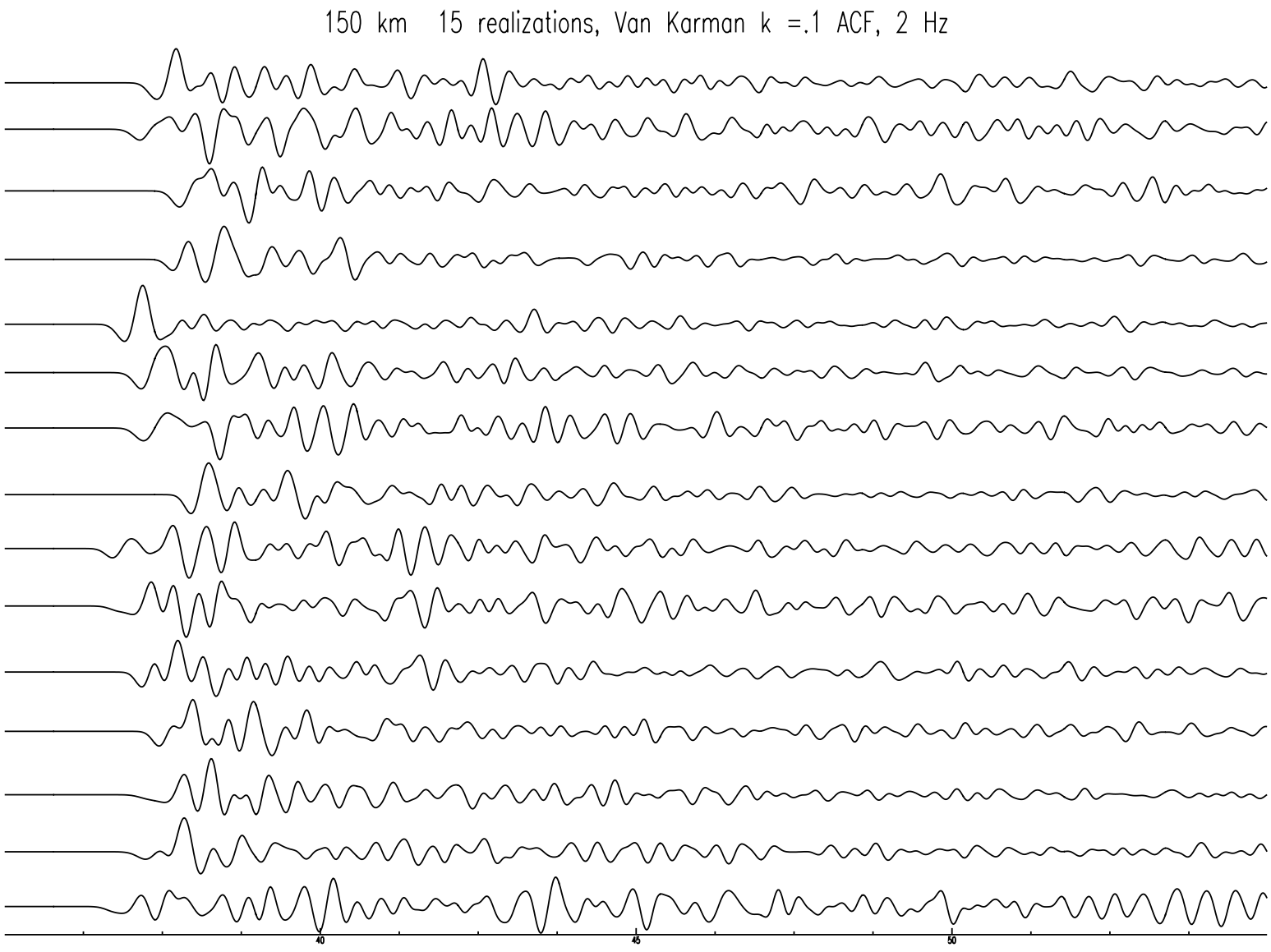

Smoothed Fnvelopes, Van Karman k 1 ACF 2 Hz, 50 runs
0.06
0.04
0.02
0.00
0
10
20
30
40
50
60
70
Note that there is no coda
at large lapse times
fehler /scratch/wp2s2/fehler/vankarman/k1/results/seisdat/envelope_all f001.sgf TueAug 1 11:06:50 2000

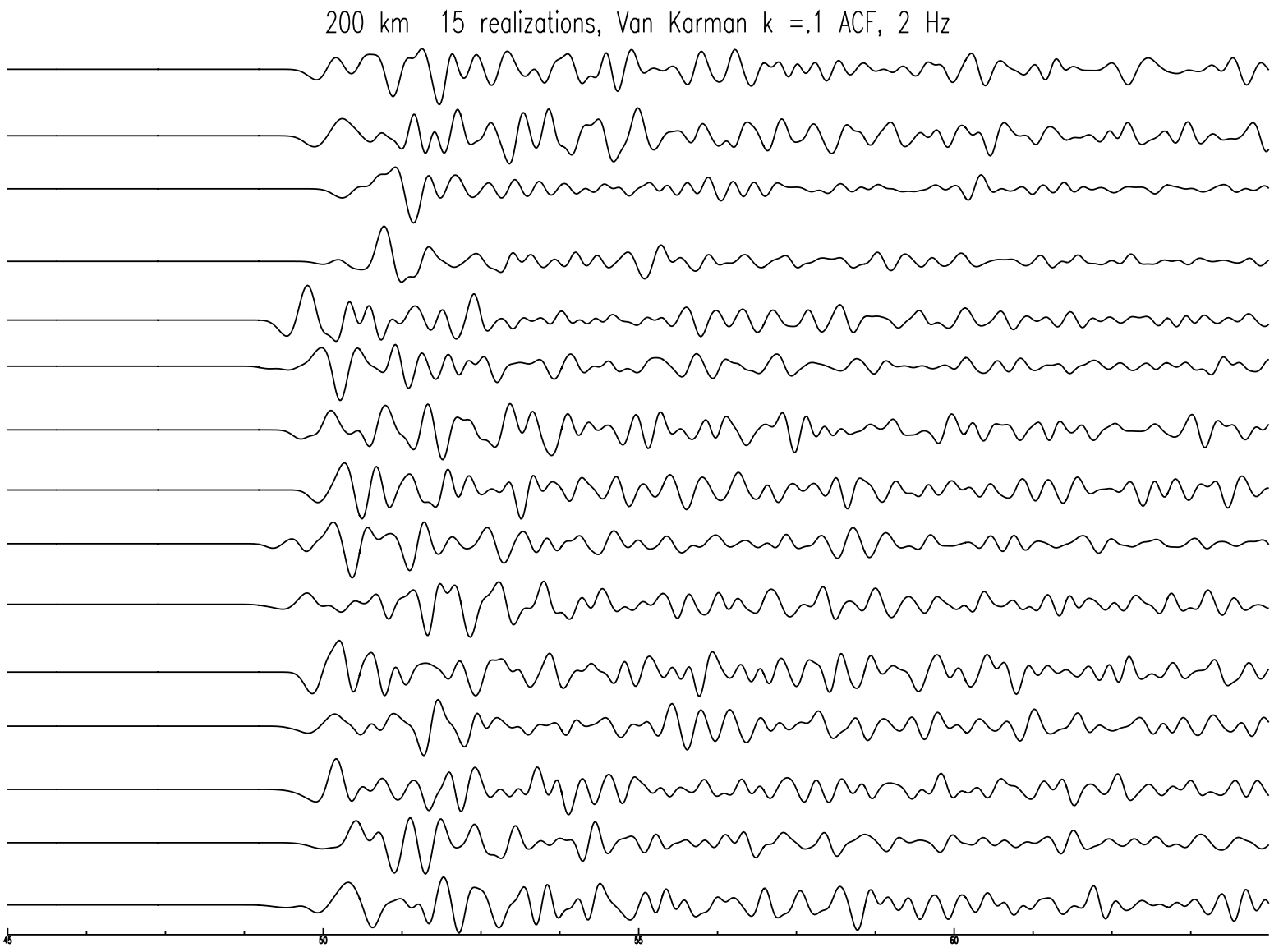

Smoothed Envelopes, Exponential ACF 2 Hz, 100 runs
0.06
0.04
0.02
0.00
0
10
20
30
40
50
60
70
fehler /scratch/wp2s2/fehler/exponential/results/seisdat/envelope_all f001.sgf Tue Mar 14 11:35:56 2000


Smoothed Envelopes, Van Karman k 0.1 ACF 2 Hz, 50 runs
2D Finite Difference Calculations
for Acoustic Wave Equation
25 Km
50 Km
100 Km
150 Km
200 Km
Note Constant Level of Coda,
Independent of Receiver Distance
0.06
0.04
0.02
0.00
0
10
20
30
40
50
60
70
fehler /scratch/wp2s2/fehler/vankarman/k.1/results/seisdat/envelope_all f001.sgf Tue Aug 1 10:34:00 2000

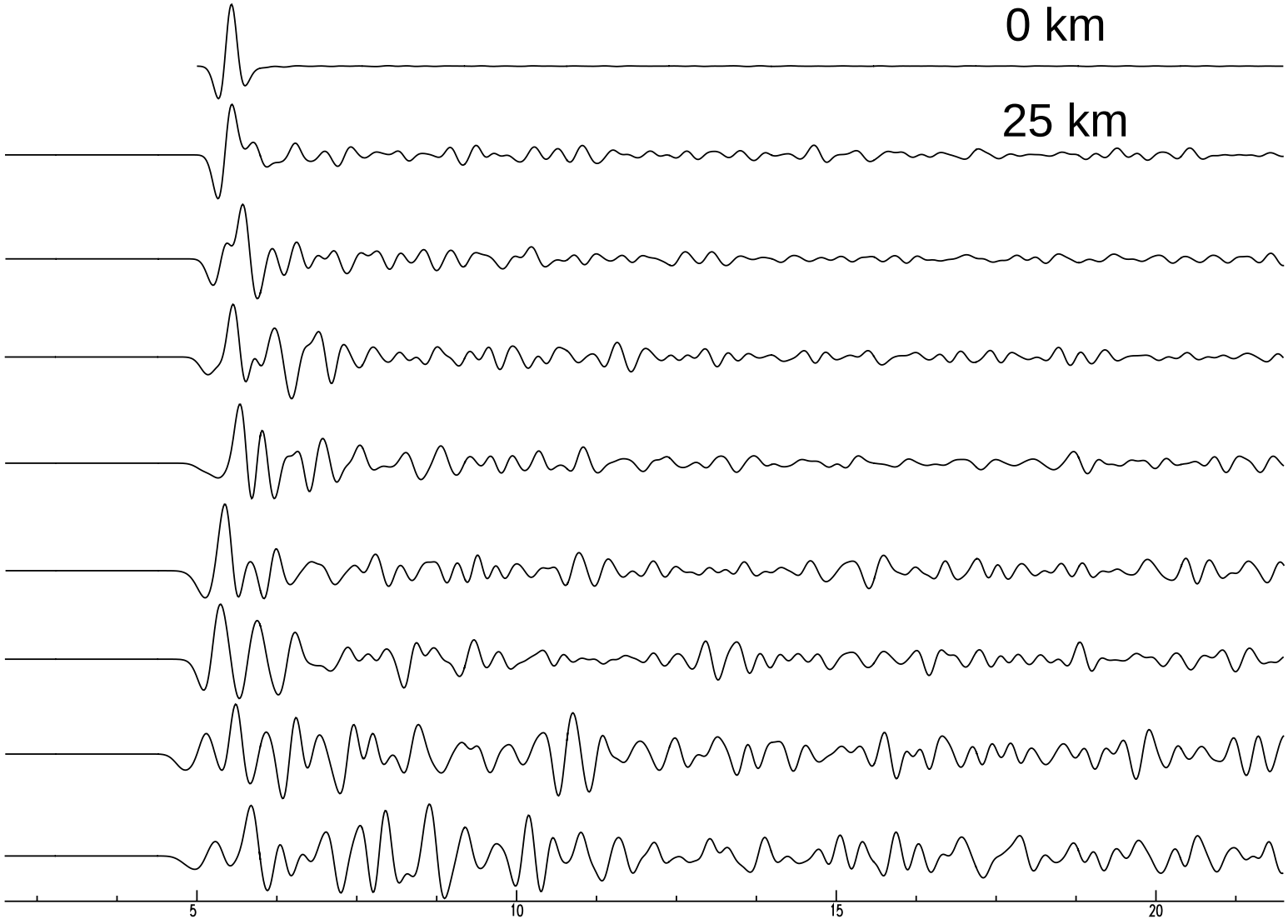

50 km 15 realizations, Van Karman k =.1 ACF, 2 Hz, Reduced Time
10
15
20
25

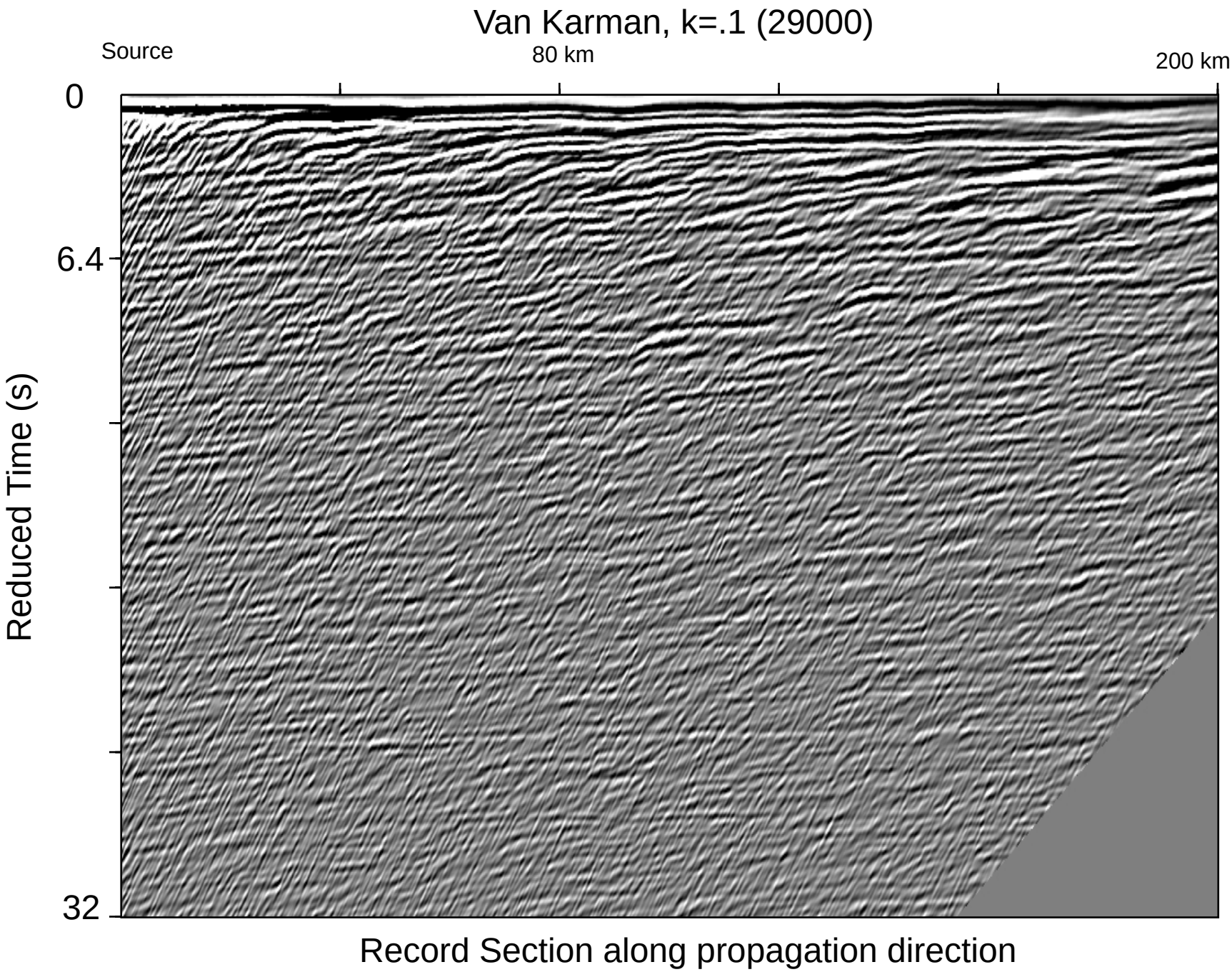

100 km 15 realizations, Van Karman k =.1 ACF, 2 Hz,
20
25
30
35

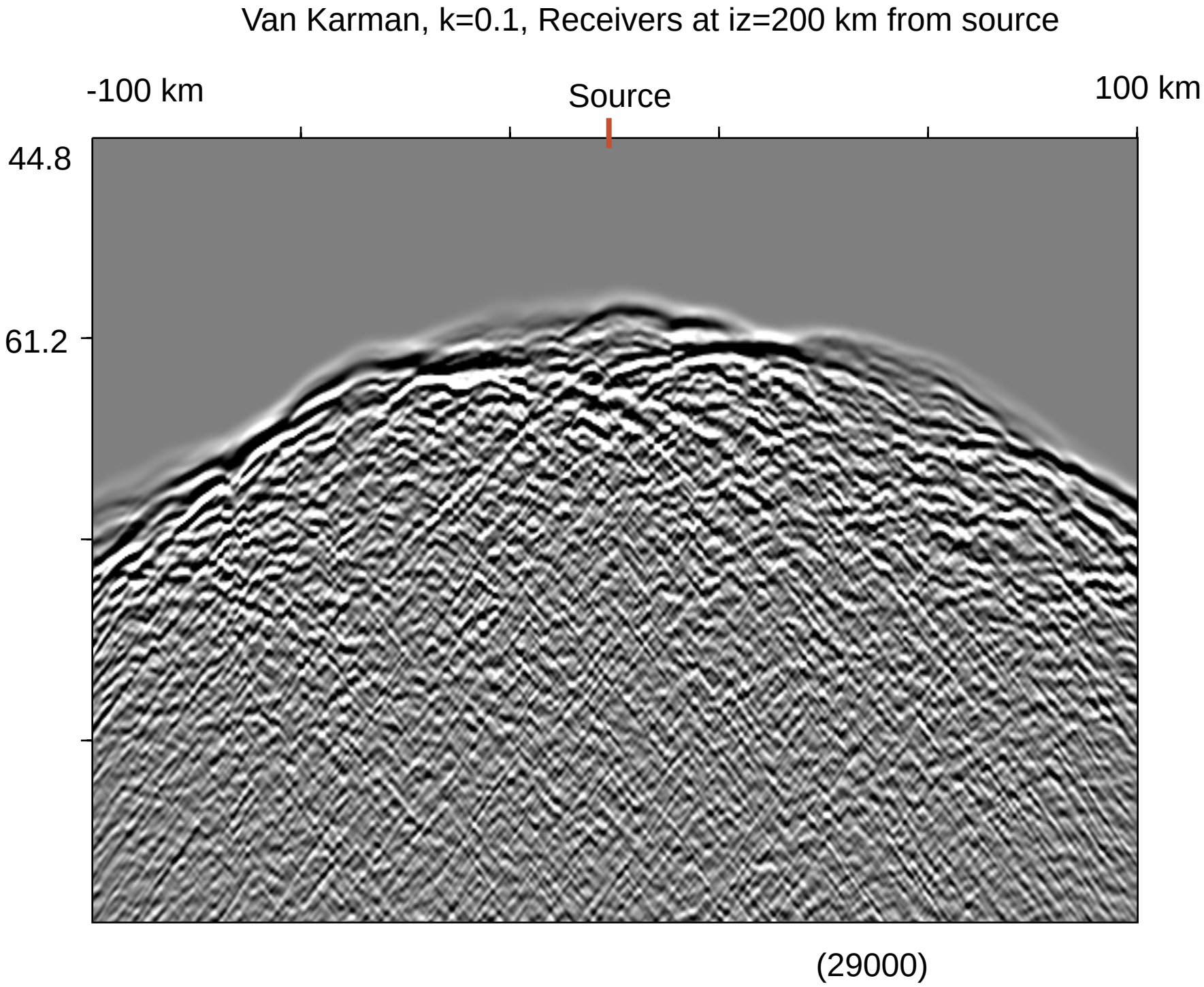

150 km 15 realizations, Van Karman k =.1 ACF, 2 Hz
40
45
50

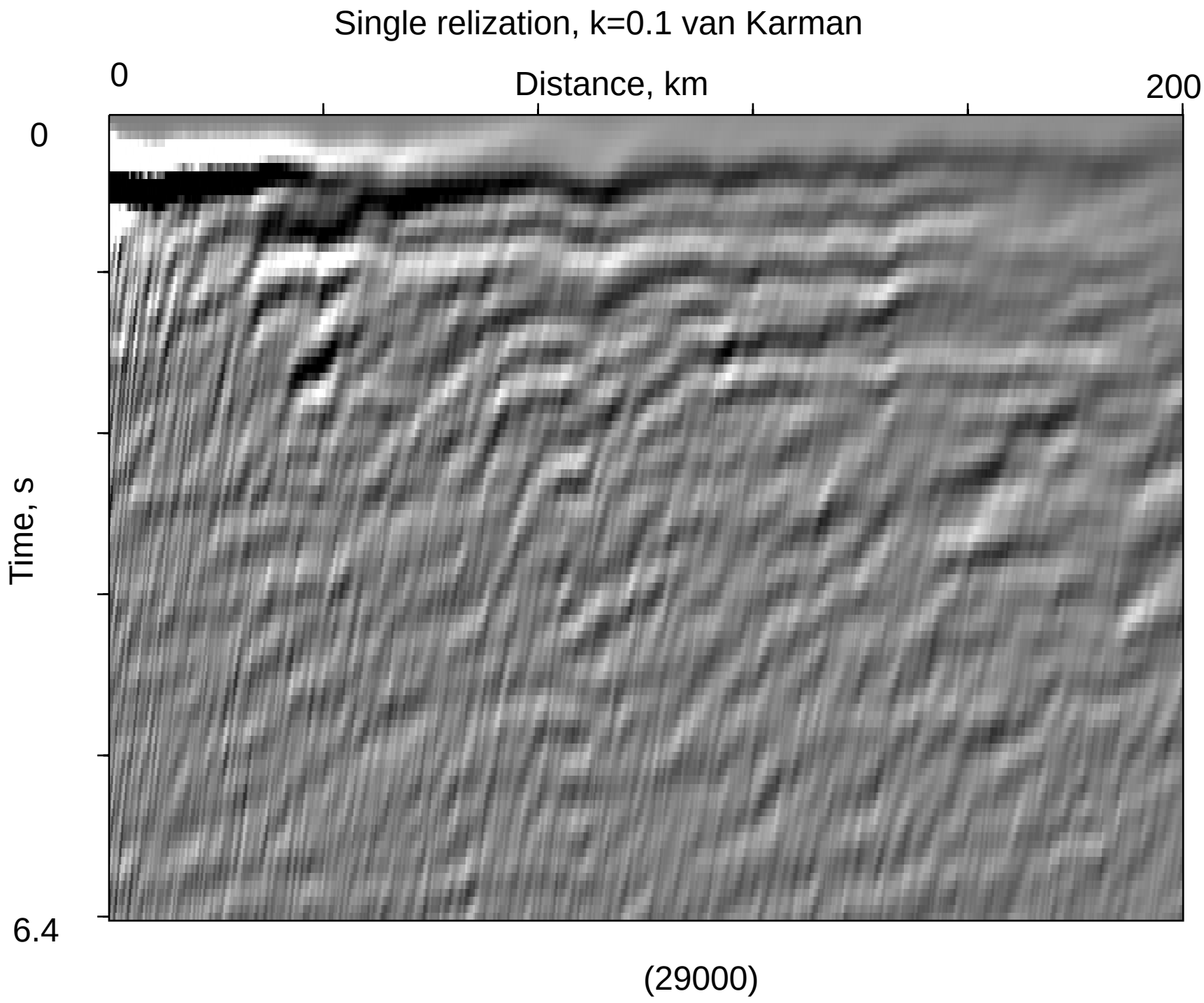

200 km 15 realizations, Van Karman k =.1 ACF, 2 Hz
45
50
55
60

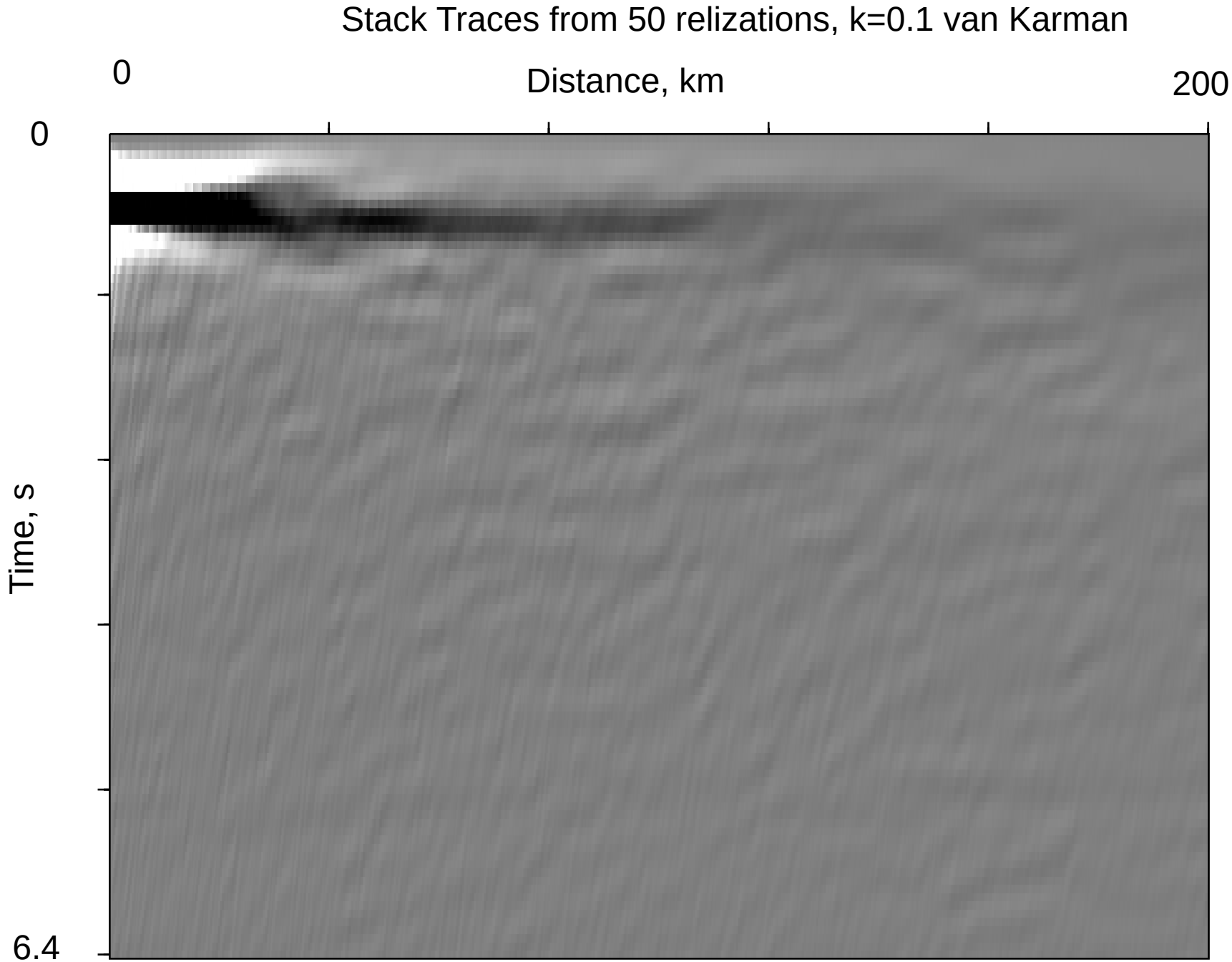
Record Section k = 0.1



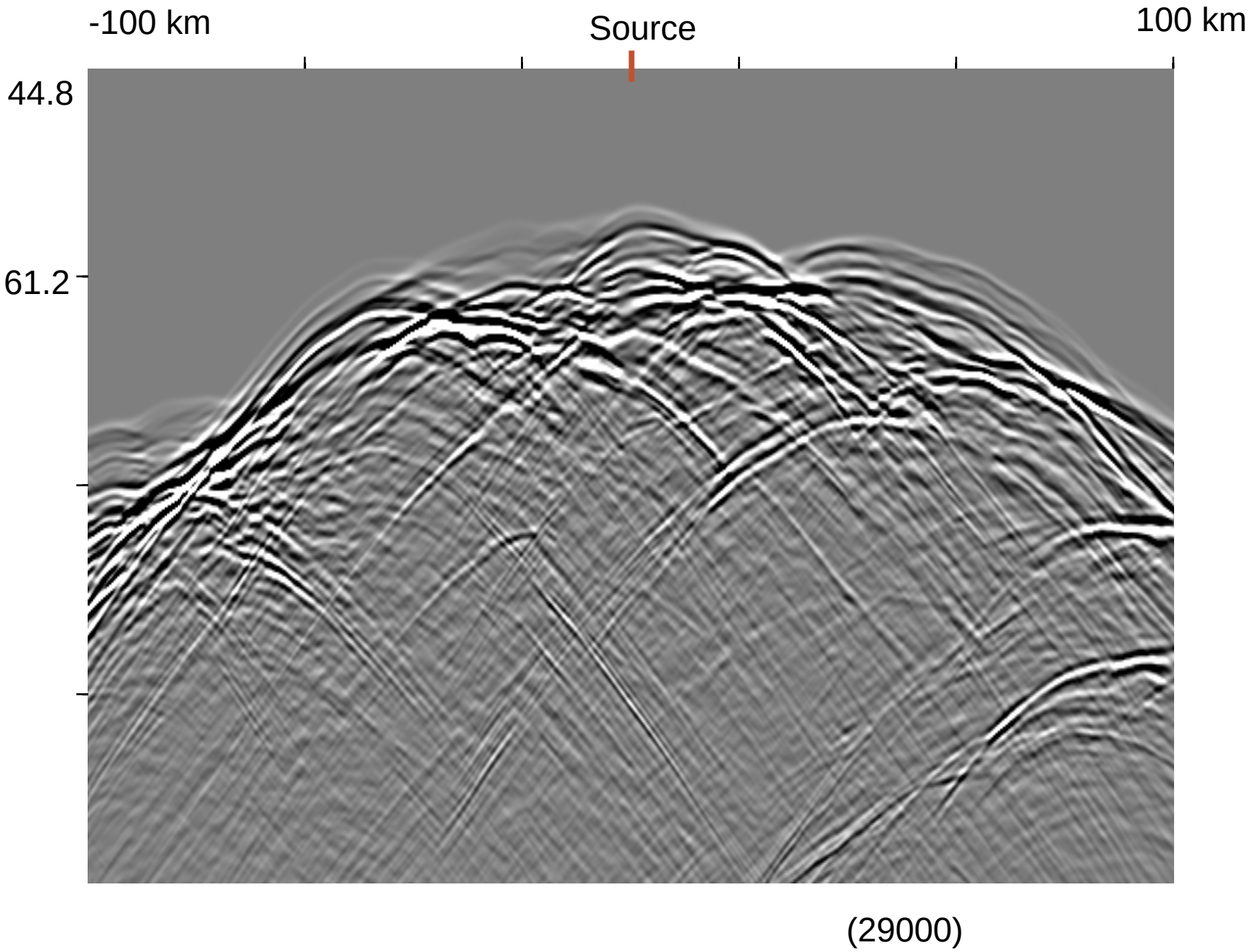

Van Karman, k=.1 (29000)
Source
80 km
200 km
0
6.4
32
Reduced Time (s)
Record Section along propagation direction

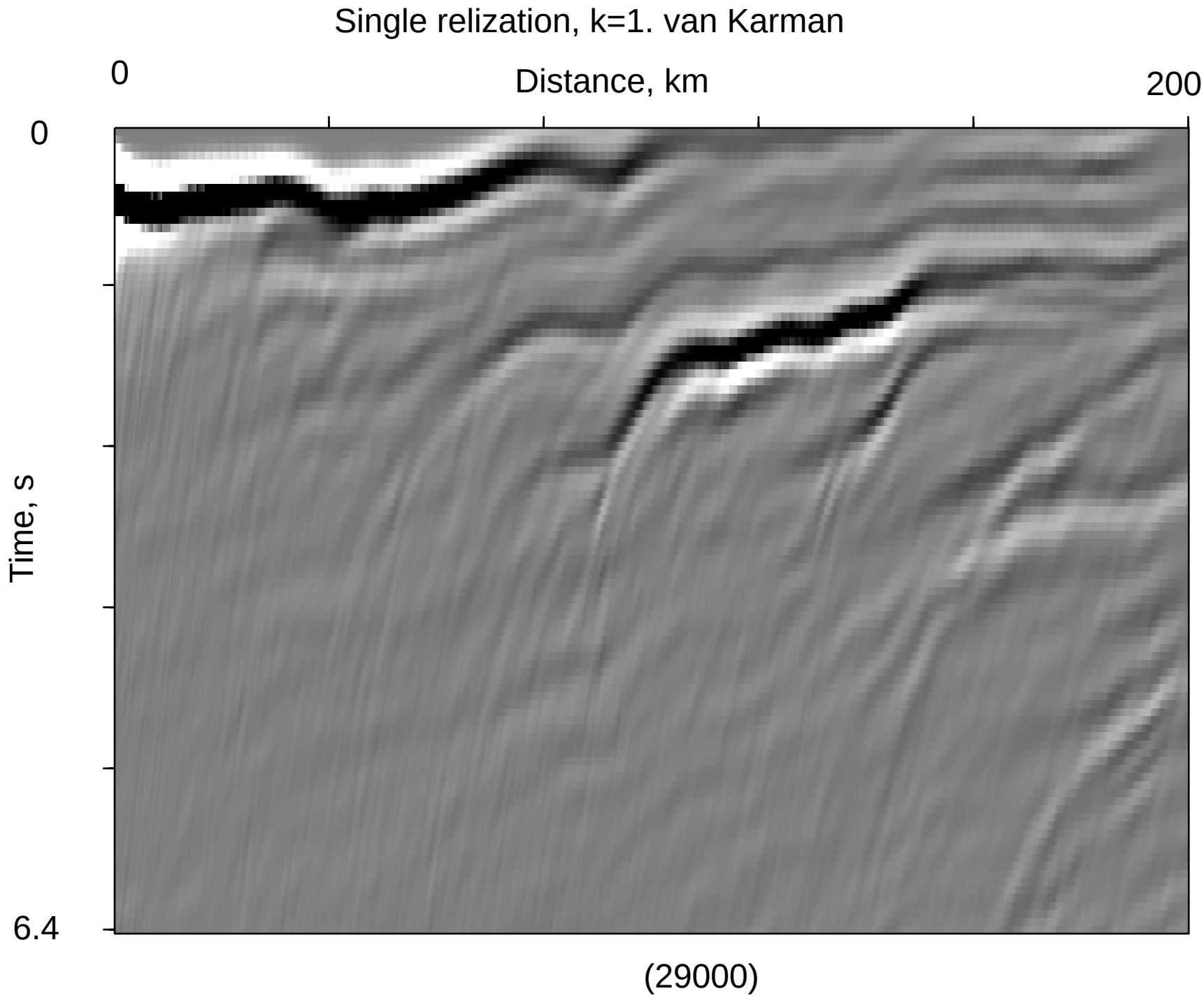

Van Karman, k=0.1, Receivers at iz=200 km from source
-100 km
Source
100 km
44.8
61.2
(29000)

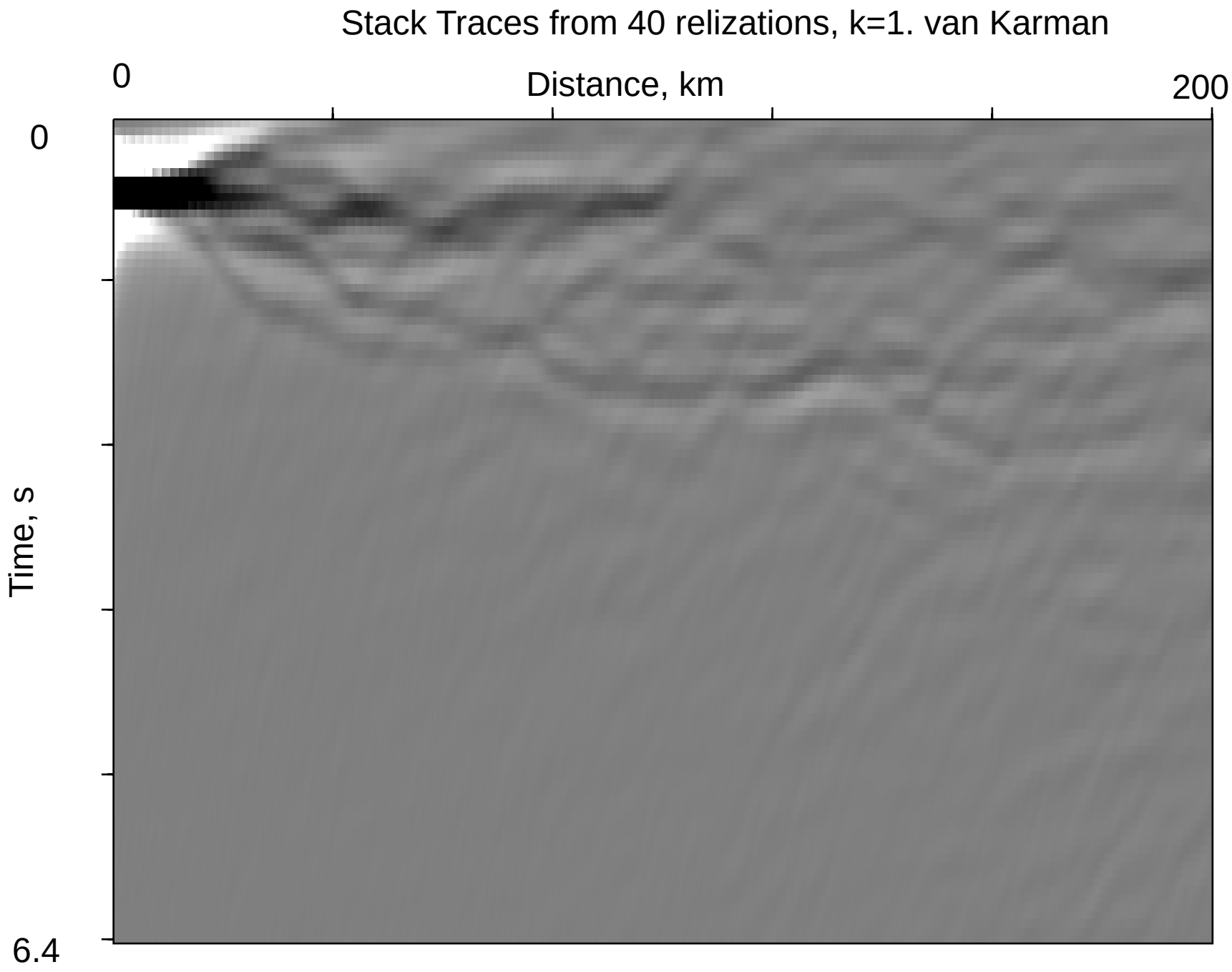

Single relization, k=0.1 van Karman
0
Distance, km
200
0
Time, s
6.4
(29000)

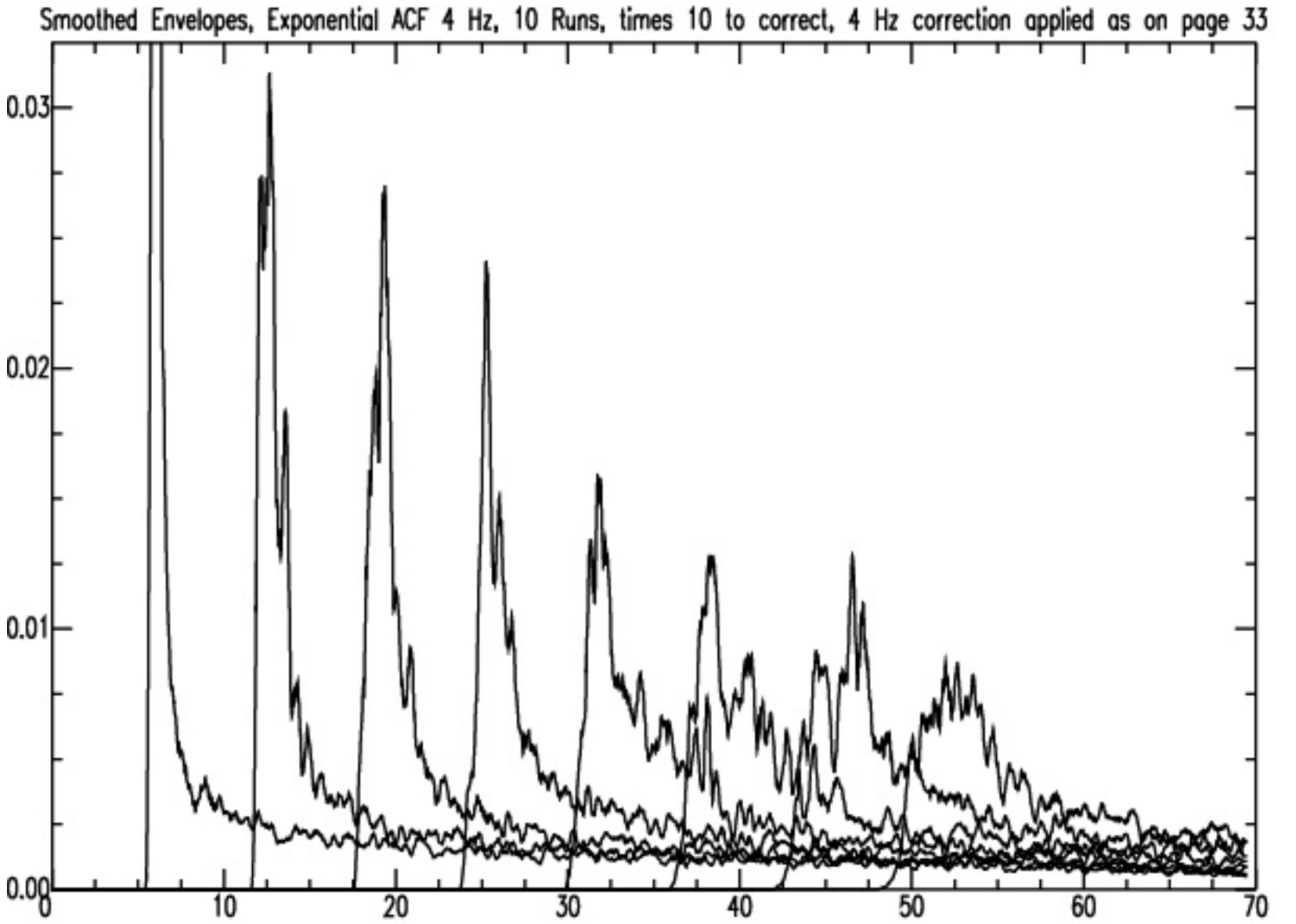

Stack Traces from 50 relizations, k=0.1 van Karman
0
Distance, km
200
0
Time, s
6.4

Van Karman, k=1., Receivers at iz=200 km from source



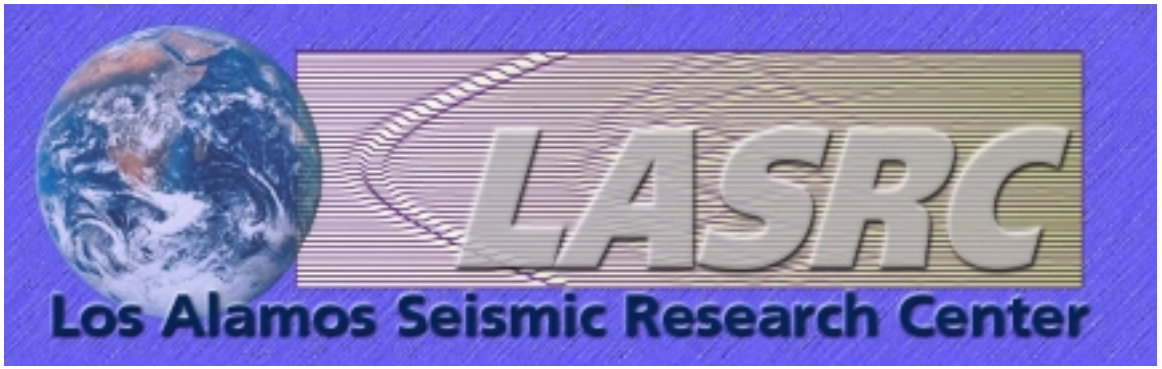

Single relization, k=1. van Karman
0
Distance, km
200
0
Time, s
6.4
(29000)


Stack Traces from 40 relizations, k=1. van Karman
0
Distance, km
200
0
Time, s
6.4


Smoothed Envelopes, Exponential ACF 4 Hz, 10 Runs, times 10 to correct, 4 Hz correction applied as on page 33
0.03
0.02
0.01
0.00
0
10
20
30
40
50
60
70
fehler
/scratch/wp1s2/fehler/exponential/4Hz/node1/results/seisdat/envelope_all
Wed Mar 15 08:45:24 2000

Conclusions

- At close range, waveforms vary mostly due to traveltime fluctuation; at large range, scattering becomes important
- Good Agreement of Markov and Extended Local Rytov Results
- Finite Difference Results are a little different than Approximate Results
  - Importance of wide angle scattering

Conclusions

- Backscattering from near-source and near-receiver regions is important